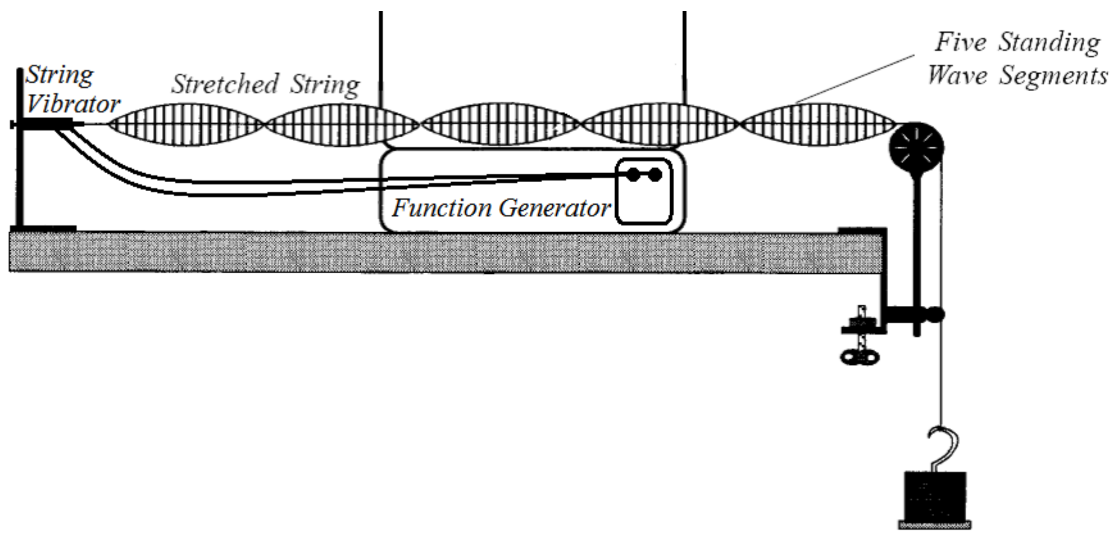
Standing Waves on a String



Produced by the Physics Staff at Collin College

Purpose

In this experiment, you will explore the relationship between string length, wavelength, frequency, linear density, and string tension in a standing wave, thus gaining an empirical understanding of the normal modes of vibration in a stretched string.

Equipment

- 1 Function Generator
- 1 Super Pulley
- 1 Slotted Mass Set w/ Hanger
- 1 String Vibrator
- 1 Ring Stand
- 1 Precision Balance
- 1 Meter Stick
- 1 Roll of White String
- 1 Mass and Hanger Set (Blue Box)
- 1 2 Banana Plug Patch Cords
- 1 Universal Table Clamp

Introduction

A wave moving within any material is evidence that energy is being transported as the result of a disturbance. There are two distinct categories of waves: mechanical and electromagnetic. Mechanical waves require some kind of material to travel in, but electromagnetic waves, including light, do not.

The speed of both categories of waves depends on two properties of the material they are moving through. For mechanical waves they are an inertial property and an elastic property. For electromagnetic waves they are the permittivity and permeability of the material. For a mechanical wave in a stretched string, the inertial property is its linear density (its mass per unit length), and the elastic property is the tension force in it.

A wave will propagate along the string if you disturb its equilibrium state at any position. When the wave reaches either end, it will reflect and propagate back toward the disturbance.

If you make the disturbance repetitive by using, say, an electric vibrator at one end, the waves propagating away from the vibrator interfere with those that are reflected back from the other end. If the length of the string is an integral multiple of the wavelength of the interfering waves, the interference pattern will be stationary in the string. Such a stationary wave pattern is called a *standing wave*.

In this experiment, you will create standing waves in a stretched string and then measure their wavelength. You will explore the relationship between string length, wavelength, frequency, linear density, and string tension in a standing wave, thus gaining an empirical understanding of the normal modes of vibration in a stretched string.

You will compare your measurements of standing waves to the theory that relates these properties. When you are finished, you will be able to

1. Explain how standing waves are created.
2. Identify the nodes and antinodes and the number of segments in a standing wave.
3. Discuss the factors that determine the natural frequencies of a vibrating string.

Theory

The properties that characterize a wave are its wavelength λ, its frequency of oscillation f (measured in hertz, or 1/s = s^{-1}), and its speed v. These properties are related by the equation:

$\lambda f = v$

Mechanical waves propagate through a medium in either a longitudinal or a transverse mode. In a longitudinal wave, each particle in the medium oscillates in the same direction as the wave propagation. Waves in a vibrating slinky spring and sound waves in any material travel in this manner. In transverse waves, each particle oscillates perpendicular to the direction of wave propagation. The waves in a stretched string vibrate in a traverse mode.

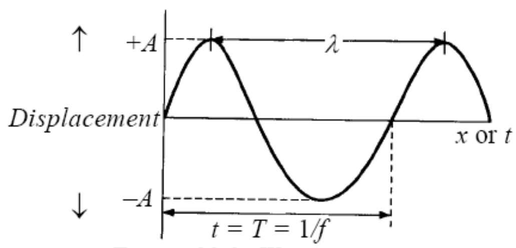


Figure 13.1. Wave properties

As each particle oscillates, its maximum displacement up and down is called the wave's *amplitude*, designated as $+A$ or $–A$. Figure 13.1 is a plot of displacement vs. either position or time. The energy being carried by the wave is related to its amplitude. The period of oscillation is inversely related to the frequency $T = 1/f$.

Two waves meeting each other will interfere. The combined wave they produce is a simple superposition of the two waves. If two waves moving in opposite directions have the same amplitude and frequency, their interference produces a standing wave as shown in Fig. 13.2. The positions of minimum displacement (destructive interference) are called *nodes,* and the positions of maximum displacement (constructive interference) are called *antinodes*. The length of one segment of the standing wave is equal to one-half its wavelength.

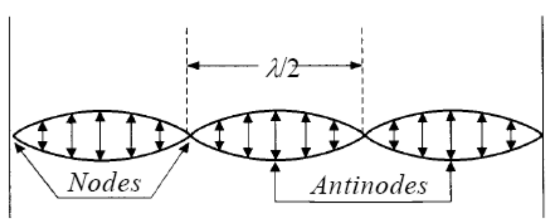


Figure 13.2. Standing wave

When a string is vibrated at one end, waves traveling from the vibrator interfere with waves reflected from the opposite fixed end. This interference produces a standing wave in the string at specific frequencies that depend on the string's density, tension, and length. If the string is vibrated at multiples of this frequency, standing waves with multiple segments will appear. The higher frequencies are known as *harmonics* (see Figure 13.3).

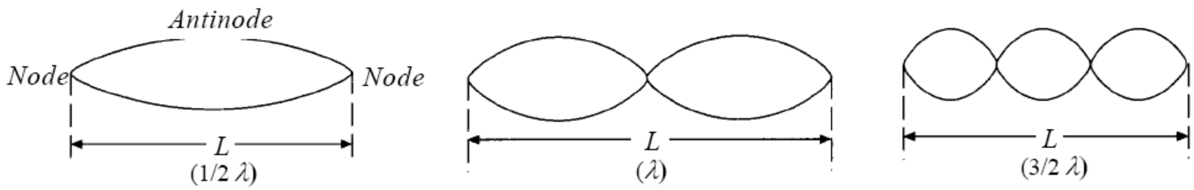


Figure 13.3. Standing wave harmonics

Note that each segment is equal to one-half of a wavelength. Thus, for a given harmonic, the wave-length becomes:

$$\lambda = \frac{2L}{n}$$

where L is the string length and n is the number of segments. You can therefore express the velocity of a wave in a stretched string as:

$$v = \frac{2Lf}{n}$$

You can also find the velocity of a wave in a stretched string from the relationship:

$$v = \sqrt{\frac{F_T}{\mu}}$$

where the tension force F_T is the elastic property in the string and the linear density μ is the inertial property. You can find the value of μ by weighing a known length of string.

$$\mu = \frac{\text{mass}}{\text{length}}$$

You can solve for the tension force by eliminating v between the two equations above:

$$F_T = \frac{4L^2 f^2 \mu}{n^2}$$

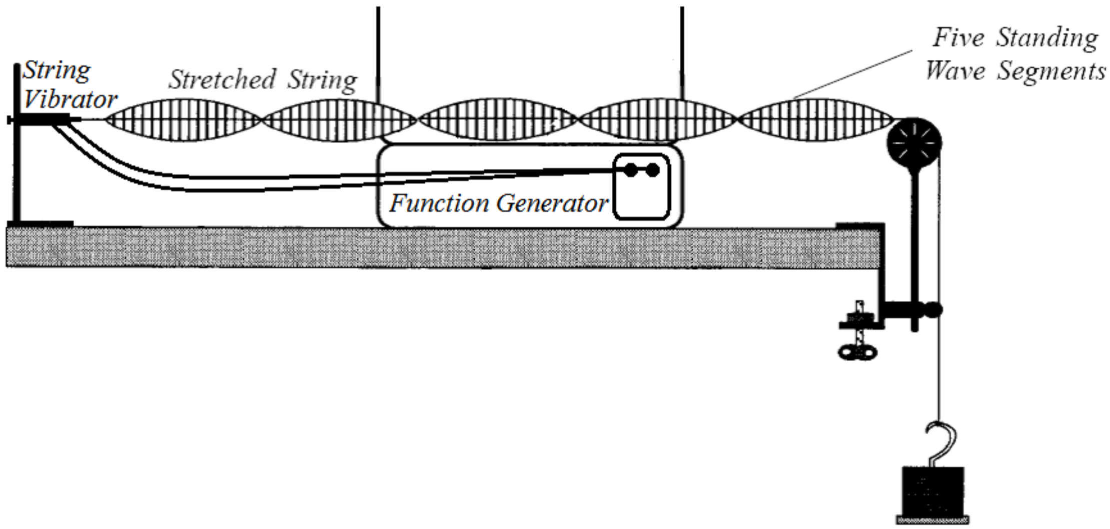
If you keep the length and frequency constant but allow the tension to vary, a graph of F_T versus $1/n^2$ yields a straight line whose slope is the numerator of this equation: $4L^2f^2\mu$. Knowing the length and frequency, you can then find the value of μ from the slope of this graph. You can also solve the last equation for the frequency:

$$f = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

A graph of f vs. n will yield a straight line whose slope is the coefficient of n: $\frac{1}{2L}\sqrt{\frac{F_T}{\mu}}$.

Procedure

You will apply tension to a length of string by hanging mass from it over a pulley, as shown in the figure below. You will then create waves in the string with a vibrator and adjust the tension (in Part A) and the frequency (in Part B) to create standing waves having from 1 to 7 segments.



1. Cut a piece of string about 5 m long. Stretch it out on the table (doubling it if necessary) and measure its length l_s (to 3 SD). Wad the string up and carefully measure its mass m_s (to 3 SD) on the electronic balance in the lab prep area. Record both values in Data Table 1.1.
2. Calculate the string's linear density μ and record its value in Table 1.1.
3. Cut about a 1.5-m piece of your string. Tie one end to the metal piece on the *String Vibrator*. Tie a loop in the other end and pass the string over a pulley that is clamped to the end of the lab table. (NOTE: The tables are narrow, so you will need to work long ways or diagonally on the table!)
4. Attach the *String Vibrator* to a ring stand. Slide the *String Vibrator* down the ring stand until the string is parallel to the table while resting on the pulley. Use banana-plug patch cords to connect the *String Vibrator* to the output of the function generator.

A. Variable Tension, Constant Frequency and Length

1. Move the ring stand so that the distance between the end of the *String Vibrator* and the top of the pulley is approximately 1.2 m.
2. Measure the length of string L between the end of vibrator and the top of the pulley. Record this length (to 3 sig. dig.) under Table 1.2.
3. Turn on the *Function Generator.* Set the wave type to a sine wave with a frequency f of 60.000 Hz. Then set the amplitude to 2.0 V to begin with. You may need to adjust the amplitude up or down throughout the experiment in order to get a clear standing wave, but you should keep the voltage under 5 V.
4. Pull down hard enough on the hanger to make the string vibrate in its fundamental mode (one segment). Now put enough mass on the hanger to maintain that string tension. Adjust the mass to *maximize* the central antinode and make nodes at the two ends that are very dark and clean (not vibrating). Record the hanging mass m_h (to 3

SD), including the mass of the hanger, in Table 1.2. Calculate and record the tension force F_T in the string.

5. Now remove enough mass from the hanger to create maximum-amplitude standing waves at each of its higher harmonics (2 to 7 segments). Record each mass and tension (to 3 SD) in Table 1.2. (NOTE: You will probably need to start using the 5 g hanger in the Mass & Hangar Set at three or four segments. It may be difficult to get 6 or 7 segments, but try to get to at least 5 segments before continuing to Part B.)
6. Plot a graph of the tension F_T vs. $1/n^2$ and draw a best-fit straight line through the data points. Calculate the slope of the line (to 3 SD), and record its value in Table 1.2.
7. Using this measured value of the slope, calculate the linear density of the string. Record it (to 3 SD) in Table 1.2.
8. Calculate and record the percent difference between your two values of linear density.

B. Variable Frequency, Constant Tension and Length

1. Put 400 g total on the mass hanger. Calculate and record the resulting tension (to 3 SD) in Table 1.3.
2. Vary the output frequency f of the function generator until you find the fundamental frequency (a standing wave having one segment and of the largest amplitude). Record this frequency (to 3 SD) in Table 1.3.
3. Find the frequencies required for the higher harmonic standing waves ($n = 2$ through $n = 7$), and record these (to 3 SD) in Table 1.3.
4. Plot a graph of f vs. n and draw a best-fit straight line through the data points.
5. Calculate the slope of the line and record it (to 3 SD) in Table 1.3.
6. Using the value of the slope, calculate the linear density of the string. Record it (to 3 SD) in Table 1.3.
7. Calculate and record the percent difference between this value of μ and your directly measured value.
8. Switch off the function generator. Return all your equipment to the lab cart and clean up your area on the lab table.