Magnetic Field of a Solenoid

\[ B = \mu n I \]

Produced by the Physics Staff at Collin College

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**Purpose**

In this experiment, you will investigate the relationship between current and magnetic field strength in solenoids. You will use this relationship to calculate the number of turns in a pair of solenoids.

**Equipment**

- Magnetic Field Sensor
- Two coils of different sizes
- DC Power Supply
- Switch
- Ammeter
- Voltmeter
- Meter Stick
- Clip leads

**Theory**

A solenoid is a helical coil constructed by winding a long wire as shown in Figure 8.1.

![Figure 8.1](image)

The magnetic field produced by the current in a solenoid is the sum of the fields due to the current in each turn. The field of a solenoid is like that of a bar magnet, with one end the north pole and the other end the south pole, depending on the direction of the current in the solenoid.

If the radius of the solenoid is small compared to its length, and if its turns are tightly packed, the solenoid is called an *ideal solenoid*. A true ideal solenoid would be infinitely long. The magnetic field inside the solenoid, near its central axis, would be uniform and parallel to the axis.

A solenoid of $N$ turns of length $L$ and carrying a current $I$ would produce a magnetic field inside whose magnitude is

$$B = \mu_0 \frac{N}{L} I = \mu_0 n I$$

and whose direction is parallel to the axis. The symbol $\mu_0 = 4\pi \times 10^{-7}$ T m/A is a universal constant called the permeability of free space. Note that the magnetic field depends only on the number of turns per unit length $n$ and the current $I$ in the solenoid. The field at any inside
point does not depend on the position of the point, i.e., \( B \) is uniform. A real solenoid is a good approximation of an ideal one for points not too close to the ends.

To study the distribution of magnetic field lines inside and around a long solenoid, consider a section of the solenoid consisting of a few turns carrying a counter-clockwise current, as shown in Figure 8.2.

![Figure 8.2](image)

For points very close to the wire in a turn, for example point \( P \) at upper left, the wire behaves magnetically almost like a long straight wire, and the magnetic field lines are almost concentric circles. At points between adjacent wires (at the cylindrical surface), the field tends to cancel. At points near the solenoid axis (reasonably far from the wires), the field is approximately uniform and parallel to the axis. The field at points outside the solenoid is relatively weak; in the limiting case of an ideal solenoid, it is zero.

The direction of \( B \) inside the solenoid is given by the following right-hand rule: Grasp the solenoid with your right hand with your fingers in the direction of the current in the windings. Your extended right thumb then points in the direction of the magnetic field inside the solenoid.

You can use Ampere's law to calculate the magnetic field inside a solenoid. If the coils are very closely spaced, the field inside will be parallel to the axis except at the ends.

Ampere's law is

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}
\]

Equation 8.1

We can apply the path integral to the rectangular loop \( a b c d \) in Figure 8.3, far from either end where \( B \) is uniform inside the solenoid and almost zero outside it.
First, calculate the closed integral in four segments—the four sides of the rectangle a b c d. The left side of Equation 8.1 becomes:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \int_{a}^{b} \mathbf{B} \cdot d\mathbf{s} + \int_{b}^{c} \mathbf{B} \cdot d\mathbf{s} + \int_{c}^{d} \mathbf{B} \cdot d\mathbf{s} + \int_{d}^{a} \mathbf{B} \cdot d\mathbf{s}$$  \hspace{1cm} \text{Equation 8.2}

Since $B$ outside the solenoid is so small as to be negligible compared to its value inside, the first integral on the right side of Equation 8.2 will be zero. What's more, $B$ is perpendicular to the line segments $b c$ and $d a$ that pass through the cylindrical surface, so the second and fourth integrals are also zero. So Equation 8.1 therefore reduces to

$$BL = \mu_0 I_{\text{enclosed}} \hspace{1cm} \text{Equation 8.3}$$

where $B$ is the magnetic field inside the solenoid and $L$ is the arbitrary length of the path $c d$. Now determine $I_{\text{enclosed}}$ for the right side of Equation 8.3.

If a current $I$ is in one of the turns, the total current enclosed by the path $a b c d$ is $NL$, where $N$ is the number of turns in the distance $L$. Thus, Ampere's law gives the magnitude of the field inside the solenoid:

$$B = \mu_0 \frac{N}{L} I \hspace{1cm} \text{or} \hspace{1cm} B = \mu_0 n I \hspace{1cm} \text{Equation 8.4}$$

where $n$ is the number of turns per unit length, $n = N/L$. 

![Figure 8.3](image-url)
**Procedure**

**Magnetic Field Sensor Operation**

The magnetic field sensor can measure magnetic fields as small as 10 gauss (G) to as large as 2 kilogauss (kG). This range covers the typical refrigerator magnet (about 200 G) and the typical lab magnet (about 2000 G). The earth’s magnetic field is about 0.5 G, so it is too small for this sensor to measure.

The sensor consists of a small box with a probe and a cable with a DIN plug on the other end (Figure 8.4). Two small Hall-effect magnetic field sensors are installed at right angles to each other in the tip of the probe.

This sensor can measure magnetic fields in two directions—axial or radial. The axial direction is parallel to the axis of the probe; the radial direction is perpendicular to it. However, you cannot read the two sensors simultaneously.

You can zero each sensor with the push of a button. Both are calibrated so their output voltage is 1 V when the magnetic field is 0.1 tesla (T), or 1000 gauss (1 kG); therefore, 1 mV = 1 G. When used with the Data Studio program, the sensor can be calibrated to display its measurements directly in gauss.

**Important.** Before you make field measurements with the sensor, you must let it come to thermal equilibrium. After powering it from the Interface, wait one to two minutes before measuring. The readings from the Hall-effect sensors are temperature dependent.

*Keep your hot hands off the sensor probe.*

**Choosing axial or radial**

There is a push button on the sensor box which toggles between axial and radial directions. A light emitting diode indicates which sensor is active. To change to the other sensor, push the button once.

**Note.** The sensor must be re-zeroed after each change between axial and radial.

The Hall-effect sensors are located in the probe’s tip. The axial sensor is at the very end of the probe and the radial sensor is on the side of the probe at the location indicated by the white dot.

**Zeroing the Sensor**

When no magnetic field is present (other than the Earth’s field), zero the sensor by pushing the **Zero** button on the box.

**Note.** Owing to the 5 mV minimum resolution of the Interface, the reading from the sensor will never be exactly zero, even when the field being measured is zero.
A. Equipment Setup
1. Plug the sensor cable into Analog Channel A on the Interface. Switch on the computer and the Interface.
2. Open the Capstone program and select the Magnetic Sensor icon. Select a Digits display and choose Magnetic Field.
3. Select one of the coils supplied. Connect the DC power supply, the switch, the ammeter, and the coil in series (Figure 8.5). For each coil you select, record the number of turns \( N \).

**Caution.** Be sure to connect the ammeter with the correct polarity.
4. Have the instructor check your circuit before you proceed.

B. Data Collection
1. Select the axial function on the magnetic field sensor. Hold the sensor away from any source of magnetic fields and zero it.
2. Close the switch and set the power supply current to 1.0 A. Record this voltage and the exact current (to 3 sig. dig.) through the circuit.

**Caution.** Do not leave the current on when you are not taking a magnetic field reading.
3. Click record at the bottom of the screen.
4. Insert the sensor probe into the coil. Move it to a point near the center of the coil. Measure and record (in Table 8.1) the radial magnetic field at the center of the coil.
5. Open the switch, remove the sensor from the coil, and click the *Stop* button.
6. Repeat steps 1-5 for currents of 2.0 A, 3.0 A, 4.0 A and 5.0 A. **Do not allow the current in the circuit to exceed 5 A.**
7. Measure and record the length \( L \) of the coil.
8. Repeat steps 1–7 for a second coil. Be sure to record your data for the second coil in Table 8.2.

C. Data Analysis
1. For each coil, plot a graph of magnetic field magnitude \( B \) vs. current \( I \). From the slope of your graph, calculate and record the number of turns \( N \) in the coil (round to the nearest integer).
2. For each coil, calculate and record the percent difference between your calculated value of \( N \) and the value printed on the coil.