Pythagórean Philosophy

Logos in the First Philosophy

Although up to this point our focus has been upon the logical nature of language, Logos as language is not the only philosophical development to derive from the original Greek notion of ‘lego’. In addition to those changes that transformed ‘lego’ to ‘logos’ *as language and Logic*, other shifts in meanings occurred as well, as Figure 2-1, introduced at the beginning of this chapter, shows. In particular, we see on the *left* descending branch of Figure 2-1 a second set of words and ideas that also derive from the root ‘lego’. Here we start from the meaning ‘calculate’, which is of course a natural step from the idea of mere counting, and from this we can easily progress to a particular brand of calculation—that of determining a ratio.[1] A ratio, then, which is a type of comparison between two numbers (like the ratio of 3 to 9, written as 3:9 by the Greeks but as 3/9 in modern notation), was yet another idea derived from the root word ‘lego’, and the Greeks accordingly called this type of comparative relation a logos.

As early Greek Science and Philosophy developed, this notion of a Logos or ratio became increasingly important; and in fact once they had the notion of a numerical ratio the Greeks were able to make the natural progression from this simple notion to the notion of a ratio of ratios. This slightly more complex relation compares the similarity of two ratios, such as 2 : 3 and 6 : 9, which the Greeks wrote as 2 : 3 :: 6 : 9 and read as “2 is to 3 as 6 is to 9. To denote this form of comparison, then, the Greeks used the phrase “ana logos”, which means “according to a ratio”. Today we call this higher order ratio by the Latin phrase, *proportion*, and as we will see directly, ratios and proportions are fundamental to early Greek mathematical philosophy. Eventually, not long after its introduction as a type of mathematical comparison in fact, this notion was generalized so as to apply to *any* comparison of similarities between different situations, and Greek phrase ‘ana logos’ was soon reduced to the single word ‘analogia’. In this way, once it had passed into general usage, the Greeks gave birth to the explicit concept of Analogy itself, a concept that—along with the concept of Metonymy, as we have seen—threads its way inextricably through so much of our philosophical studies. This more general application of the term ‘analogy’ notwithstanding, however, it is the *original* meaning of ‘analogos’, that of a *mathematical analogy*—a numerical proportion, such as the proportion between 6/9 and 8/12, in which the two ratios are both 2-to-3 (or 2/3)—that constitutes our focus here. And although the two concepts of ratio and proportion are fairly simple as mathematical notions go, by using even such simple ideas to form the basis for their primitive Logic the Greeks were in a position to develop scientific and philosophical theories that were genuinely *mathematical*. And as it just so happens, the first theory of this type ever developed, the first full-blown mathematical scientific theory that the Greeks ever came up with, was a theory of musical harmony. This Greek theory of musical proportions, then, marks the very first time in the recorded history of the West that a scientific theory—a theory about the real world—was founded upon both Logical and mathematical principles.

The first philosophers in the West to investigate the various implications of the concept of 'logos' as "a ratio between numbers" were almost certainly the members of the peculiar sect known as the Pythagóreans. This organization—at once religious and philosophical, and named for it's founder and original leader, the philosopher Pythagóras of Samos—consisted of Greek intellectuals who lived in the Greek colonies of southern Italy. Their main interest, apart from the tenets of their Orphic religion (which need not concern us here), was what we today call mathematics. In fact, our word 'mathematics' comes to us through the Pythagóreans, for the Brotherhood itself was divided into two main groups: the Akousmatikoi or Auditors, who were allowed to "hear" the Pythagórean teachings; and the Mathematikoi or Mathematicians, who actively investigated Pythagórean teachings, viewed as at once religious and mathematical mysteries, and taught these mysteries in their lessons (in Greek, 'máthemai' = 'lesson'). Unfortunately, this is about as much as we know about the Pythagórean organization; and among the historical documents that have come down to us from antiquity, no trace remains of anything written originally by Pythagóras himself. We do however posses many quotations that are at least reportedly from Pythagóras, as well as many more-or-less late biographies of his life and teachings. In addition, a large number of original documents written by some of the sage's more productive successors and their followers have managed to survive. And these extant literary remains contain enough information to allow us to construct an adequate understanding of the Pythagórean mathematical Philosophy. What we find their is absolutely amazing.[2]

To begin with, we note that the philosopher Pythagóras is believed to have been born around the year 570 BCE on the island of Samos, an island that lies off the coast of what historians call Asia Minor but what is now the country of Turkey. To the Greeks of Pythagóras' time the west coast of Turkey was known as Ionia, and even before Pythagóras was born Ionia had seen the development of the earliest Western sciences. Although we now call Tháles, Anaxímandros, and Anaximénes—the men who invented these sciences—Ionian *philosophers*, their true intellectual achievement was actually more akin to Science than to Philosophy. Thanks to these men, Science had emerged in the Ionian city of Miletus only some twenty years before Pythagóras' birth, beginning with the development of Tháles scientific theories about the origin and structure of the World—or "The All", as the Greeks called it at the time. Tháles' theories must have attracted some interest, for later Greek philosophers and historians still recognize his genius, and tell us that Anaxímandros was in fact his student. Whether this is true, and whether in fact Anaximénes was the pupil of Anaxímandros, it is obvious that the Milesian school of scientists caused quite a stir. And from the multiplicity of schools that quickly arose in philosophical opposition to this Milesian school we may surmise that interest in the subject spread rapidly throughout Greece. As a result, we may assume that Pythagóras was familiar with the teachings of the three main Ionian "philosophers"; and in fact one of the things we read in the (more-or-less reliable) bibliographies of Pythagóras is that he learned basic mathematics from the very first Ionian scientist, Tháles himself. At any rate, the philosophy of the Pythagóreans certainly shows traces of Ionian influence, and although this may be attributable to the general influence of the Ionians, it might also suggest that, as the existing stories relate, Pythagóras received some direct intellectual training in Ionia. Eventually, as the story goes, when Pythagóras was about forty years old the political situation in Samos had become unstable, and the philosopher was forced to leave his home town. After traveling about for some time, Pythagóras settled in one of the many Greek colonies of southern Italy. These colonies were known collectively to the Greeks as Magna Graeca, and there, in the city of Croton, Pythagóras formed his religious/philosophical school.

This original Pythagórean society met regularly in a meeting-house and, strangely enough, investigated the mysteries of both the religious teachings of Orpheus and the philosophical implications of mathematics. In time, word of Pythagóras' teachings spread, and before too long Pythagórean "brotherhoods" were being established in several other cities around southern Italy. Unfortunately, along with their religious and philosophical activities the Pythagóreans also allowed themselves to become embroiled in the political machinations of Magna Graeca, and when an opposing party came to power the several Pythagórean schools involved were destroyed and many members of the Brotherhood were slain. After this, the remaining Pythagóreans spread out across Greece and settled in different cities; some of them continued to write and teach, and it is from their writings, for the most part, that we learn of the life and teachings of Pythagóras and the Pythagóreans.

The Harmony of the Kosmos

Like the Ionian philosophers before him, Pythagóras seems to have wanted to explain the world in as few terms as possible, perhaps even only one term. Tháles, the original Ionian this choice perhaps seems natural enough. After all, water can easily enough be frozen into earth thinker, had chosen Water as the single substance from which all else in the world devolved, and -like ice, or vaporize into air-like mist. And since he explained everything in the World in terms of this *one* substance, Tháles was what we now call a Monist (the Greek 'mono' = 'one thing'). Anaxímandros, Tháles' student and an interesting thinker in his own right, was also a Monist, for he too posited a single fundamental element for the world. But this second Greek scientist must have realized that any specific *material* substance (such as Tháles' Water) needs boundaries in order for it to be material or substantial, and so as his primary element Anaxímandros developed the altogether insubstantial idea of what he called "the Boundless" or "the Unbound". This immaterial element, obviously enough, marks quite a departure from the material Water of Tháles. Nonetheless, since Anaxímandros used but this one single element as the physical basis of the world, he too was what we call a monist, and his philosophy of the Boundless was truly a form of Ionian Monism. The Pythagóreans also embraced this earliest form of metaphysics—Monism, that is—for they too had but a single creative element at the basis of their cosmogony. In fact, Pythagóras even went so far as to use Anaxímandros' concept of the Unbound for his philosophical system, although he did not make it the fundamental element of the Kosmos, as Anaxímandros had done. Rather, the mathematically minded Pythagóras found his most fundamental substance in what the Greeks called 'aríthmos', a word that is usually translated as 'number'. As the Pythagóreans monists have it, then, everything in the world—which Pythagóras was purportedly first to call a Kosmos or Order—consists ultimately of just one thing, and that one thing is Aríthmos or Number.

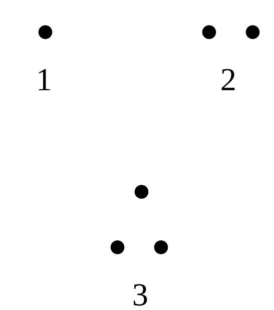
The idea that things are made of numbers sounds peculiar to us when we first hear it; but we can perhaps begin to overcome this feeling of peculiarity by realizing that the word 'aríthmos', like the word 'logos', can have more than one meaning. Of particular interest in this connection is undoubtedly the second most significant Greek word that Pythagóras used technically, the word 'harmonía'.[3] Originally 'harmonía' meant 'a fastener' or 'a joint' and even, not surprisingly, 'an agreement between people', which is a kind of abstract joint. For the Pythagóreans, though, 'harmonía' had other, philosophically significant connotations. Originally, the word had denoted the device by which strings were "fastened" to the lyre—the musical instrument of Pythagóras' god, Apollo. Before too long, however, the word 'harmonía' came to denote—perhaps by means of Metonymy—the *system* of tones to which the strings of the lyre were tuned. Accordingly, Pythagóras' Harmonía was just that specific set of *musical relationships* among the sounds of the lyre that make it harmonious, and this is just the notion we need to understand what Pythagóras meant by the related word 'aríthmos'. For the words 'aríthmos' and 'harmonía' are cognate words—that is, they come from the same ancient root: 'rim'—and so 'aríthmos', like 'harmonía', can also bear the connotative meaning of 'joint' or 'connection' in addition to its more standard, denotative meaning of 'number'. The Pythagórean Aríthmos, then, is a type of joint or fastener—a relation—but in this case the elements "fastened together" by the relation of Aríthmos are not musical tones but rather pairs of opposing concepts that are fundamental to the Pythagórean understanding of the Kosmos.

As no less an authority than Aristotle (among others) tells us, the Pythagóreans often spoke of *ten* pairs of opposites, and the Brotherhood in fact had a more-or-less official Pythagórean Table of Opposites (ten in number). These Opposites, which express qualities of what Pythagóras called the Monad and the Dyad—the One and the Two—begin in some sense with the concept of Aríthmos as number, and thus the notion that Pythagóras derived all things from Number is justified. Nevertheless, the list of Opposites includes some opposing concepts that seem inappropriate when applied to the concept of Number, but reflect instead the nature of the *physical* Kosmos. And so, although the Pythagórean philosophy of these ten pairs of Opposites begins with the idea of number, it also involves the notion of Aríthmos as a type of physical fastener or *connection* between kosmic Opposites. And it is just *this* connotation of 'aríthmos' that seems to have been significant for Pythagóras, for it allowed this mystic philosopher to see his Aríthmos as the very heart of the Harmonía of the Kosmos itself—a Harmonía that virtually generates the entire Kosmos from the interactions of the original pair of Opposites, the Monad and the Dyad. From these concepts, then, Pythagóras formulated what is for all intents and purposes a causal explanation of the origin of the Kosmos, as follows.

The first pair of Pythagórean Opposites, known as the Bound and the Unbound, are qualities or characteristics of the Monad and the Dyad respectively. And they are a philosophically significant pair, for they allow Pythagóras to introduce Anaxímandros' concept of the Unbound, while opposing to it his own idea of what he calls the Bound. These two opposite qualities, the Bound and the Unbound, emerge, as it were, from what you and I would call the numbers One and Two, which Pythagóras called the Monad and the Dyad. For Pythagóras, however, the One or Monad and the Two or Dyad are not numbers, strictly speaking, because numbers, as far as the Greeks were concerned, are increased more by multiplication than by addition. This is not the case for One and Two, as we can see quite easily. For instance, 1 x 1 = 1 but 1 + 1 = 2, and 2 x 2 = 4 but 2 + 2 = 4 also; and in all of these equations the results are either greater for addition than for multiplication, or they are the same. The first true Aríthmos, then—the first *true* Number, at least for the Greeks—can only be the Triad or 3, because as we know 3 + 3 = 6 but 3 x 3 = 9, and of course 9 is greater than 6. In contrast, the Monad and the Dyad are not numbers but are, instead, the two fundamental principles of the Kosmos, and for the Pythagóreans they are in fact hardly distinguishable from the Bound and the Unbound. Accordingly (if we are to speak proper Pythagórean, as it were), when talking about the Monad and the Dyad we must restrict ourselves to saying only that the Monad and the Dyad themselves, through their interaction or *Harmonía*, give rise to the Aríthmoi or Numbers.

The concept of the Pythagórean primal harmonía can be displayed by means of certain diagrams that represent Pythagórean ideas, and these are illustrated in the following figures. The Pythagóreans themselves are said to have used such diagrams, although they commonly represented their numbers by pebbles rather than the dots we use here. At any rate, theses

Arithmos and Arithmoi:



figures illustrates the development of Numbers from the Bound and the Unbound through the power and Harmonía of the Monad and the Dyad (displayed in the box in the upper part of the figure). As we see, this gives us not only the primal Opposites, the Monad (1) and the Dyad (2), but also the result of this Harmonía: that is, the Triad (3) itself, because 3 = 1 + 2. And this result, the odd number 3, is none other than the first of the Pythagórean Aríthmoi or Numbers, as well as the first of what the Pythagóreans called the *triangular* numbers (because of the shape of the array of dots). Continuing on in this manner, the Pythagóreans generated the next number, the even number Four—which they called the Tetrad and first *square* number (see the diagram below). By using what is called a "carpenters square"—or 'gnomon', in Greek—to combine the Triad with the Monad in the manner shown (a method used by the Pythagóreans themselves), they generated the number 4. And having done this, they now had Pythagóras' two primal elements—the Monad and the Dyad or the One and the Two—as well as the first two Pythagórean numbers—the odd number Three and the even number Four. And these first four numbers, whose sum is of course 10, the Pythagóreans considered sacred. All the same, these early mathematicians were interested in all of the numbers, and so they continue by adding the next *odd* number, Five, to the first square number, to get the next square number, Nine (as shown in the figure). And when we similarly add the next *odd* number, Seven, to the previous square number Nine, we get yet another square number, Sixteen (which is as far as we go in this diagram, on the left at any rate). Of course, *ultimately* this process—the process of adding successive *odd* numbers to each new *square* number produced—can be carried on unendingly (or unboundedly, as Pythagóras might put it), and eventually we would use *all* of the odd numbers to produce *all* of the square numbers. More significantly, however, since we start with a *side* dimension of one and increase the *side* dimension for the successive squares by one at each step (that is, 2 for the first square number, 3 for the next square number, then 4, and so on), the side dimensions of the series of squares run through the whole sequence of counting numbers in the natural order: 1, 2, 3, 4. The length of the sides of the successive square numbers thus represents the numbers 1, 2, 3, 4, and so on.

The situation is similar—though not identical—on the right-hand side of the figure, the side for *even* numbers. There, as we see from the side dimensions of each new figure, we are again starting with a side dimension of one (for the Dyad) and we again increase each side dimension by one. And so here too, as with the odd numbers, we generate the whole number sequence, 1, 2, 3, 4, and so on. There is one difference for the even numbers as opposed to the odd numbers, however, for by adding the first *even* number, Four, to the Dyad and thereby generating the next *even* number Six, we get a number that is not a *square* number, but rather what the Pythagóreans called an *oblong* or *rectangular* number. And similarly when we add this number, Six, which just happens to be the *next* even number, to its oblong-number counterpart (which we just generated), we get the *next oblong number*, Twelve. And now we can see precisely why the situation on the right-hand side of the figure is *similar* but not *identical* to that on the left-hand side; for—although both the Monad and the Dyad are productive, with each generating the entire sequence of counting numbers—in the sequence on the left we started with the *Monad*, added consecutive *odd* numbers, and produced only *square* numbers, whose two dimensions (the right and bottom sides) are always the *same* length. On the right hand side, in contrast, we started with the *Dyad* and added consecutive *even* numbers, and yet we have already generated two *different* oblong numbers. In addition, since the sides of the *squares* are always the same

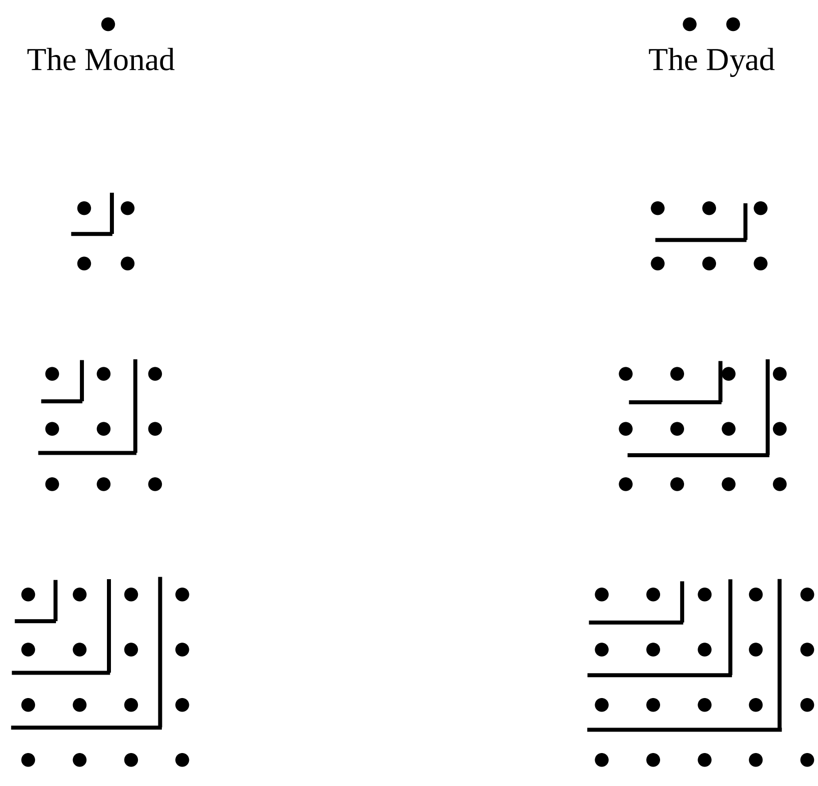


Figure 2-10. The Pythagorean System of Arithmos and Arithmétike

lengths, the ratios of these sides is always the same, for the ratio of 2-to-2 is 1, the ratio of 3-to-3 is 1, the ratio of 4-to-4 is 1, and so on for the left-hand side, with each ratio equal to the Monad. For the oblong numbers, however, the sides are not only *not the same* length, but for each oblong number they produce different ratios. That is, the first oblong number, Six, has dimensions of 2 x 3, which gives a ratio of 2-to-3, while the second oblong number, Twelve, has dimensions of 3 x 4, for which the ratio is 3-to-4, with each ratio being a type of Dyad. What is more, as we continue to produce new oblong numbers with longer sides, the ratios of these sides continue to increase, and so when we add the next consecutive even number, Eight, to the previous oblong number, Twelve (as we can see from the diagram) we get yet another oblong number, Twenty, whose side dimensions are now 4 x 5 and whose ratio is of course 4-to-5. Ultimately this generative process, like the others we have seen so far, could go on endlessly, eventually generating all of the numbers.

As the first few steps in these two infinite processes show, the Pythagórean method of Harmonía―the method of linking different kinds of numbers together in different ways―produces not only the entire number sequence itself, but also allows us to discover other various relationships (or harmonies) between numbers. The square numbers and the oblong numbers, for example, differ not only in the consistency of the former (similar squares, always) and the variability of the latter (dissimilar oblongs, always), but also in their generative potentials. The odd numbers always generate ratios of one or unity (consistency again), which gives us the notion of identity; while the even numbers generate ever-finer ratios of increasingly larger numbers (variability again), which gives us the notion of diversity. As a result, the very production of the world―this Kosmos, as Pythagóras would say―follows directly from the creative force of the harmonious interplay of the ratios that emerge from the primal elements, the Monad and the Dyad. Of course, Pythagóras would not have called these combinations of numbers *ratios*, since that is a Latin word, but rather *Logoi*, which is the plural form of the Greek word Logos. Consequently, the Pythagórean study of the creative force that follows from the harmonious interplay of "ratios" is in reality nothing but an incipient form of mathematical Logic. We shall see more of this early Logic directly.

Pythagóras epitomized this creative force in his ten Pairs of Opposites: contrary ideas which formed a series of cosmic concepts, as it were. And we know the first of these pairs (see Figure 2-10), for it is just the Bound and the Unbound principles with which we started this discussion. The second pair, as we see from the illustration, is that of the One and the Many, which represent the Bound and the Unbound in terms of number or multiplicity in general. The third pair, then, the Odd and the Even, represents the two kinds of numbers that can be generated according to the Monad and the Dyad. And these two kinds of numbers in turn generate the next pair, what are called the Square and the Oblong numbers, whose two dimensions as we have seen generate ratios that are either always one, and thus always the *same* or Equal (for Square numbers), or always different, and hence always Unequal (for Oblong numbers). And these two—Equal and Unequal—constitute the last pair of Opposites that express numerical or arithmetic ideas. From this point on the list of opposites moves out of the realm of mere arithmetic and into the realm of the physical world. The Straight and Curved, for instance, Pythagóras' next pair of Opposites, are certainly not qualities of numbers; rather, they are qualities of things in the world. And similarly, Rest and Motion, Light and Darkness, and Right and Left are themselves either physical things or qualities—characteristics—of physical things. And finally, Good and Evil, which are of course moral qualities, were also viewed by the Greeks as characteristics of the Kosmos itself. Thus, through the creative force of Aríthmos and Harmonía, these ten pairs of Opposites succeed each other, generating the Kosmos from its most *fundamental* qualities—the Bound and the Unbound—culminating in its most *sublime*—Good and Evil.

The Music of the Spheres

The beauty of the Pythagórean Kosmos—the universal Order that he elicited from the Chaos—is nowhere more wonderfully expressed than in Pythagóras' theory of musical harmony. Admittedly, Greek music must have been somewhat systematized before Pythagóras, for both the four-stringed and seven-stringed lyres (harp-like instruments) were in existence before Pythagóras was born. And Orpheus, Pythagóras' purported spiritual mentor, was highly reputed throughout Greece for his unparalleled, even god-like, musical abilities. Nevertheless, for some two thousand five hundred years now Pythagóras has been recognized as the first Greek to approach the tuning of musical instruments *mathematically*.

According to a popular myth—which at least illustrative, even if not true—Pythagóras was walking by a blacksmith's shop and noticed that when different hammers struck the anvils, different musical tones rang out. What is more, as he soon discovered, the tones from the ringing hammers were in harmony with each other only whenever the *weights* of the hammers themselves stood in simple *arithmetical ratios*, in particular, the ratios we have already seen: the sacred ratios of 1-to-2, or 2-to-3, or 3-to-4, and so on. From this chance experience, as the story goes, Pythagóras discovered the mathematical theory of musical harmony. Of course, there is no way to determine whether this story is true, given what little we actually know about Pythagóras himself; but we can nevertheless be sure that the great Orphic philosopher knew of the arithmetical ratios of the musical tones. It is generally accepted that he was the first person to investigate these musical relations or harmonies, and he did so using a device known as the monochord—a musical instrument with only one string (as illustrated at the top left of Figure 2-11). Using his monochord, then, along with his knowledge of mathematical ratios, Pythagóras apparently was able to produce the oldest mathematical theory of musical harmony of which we have record in the West.

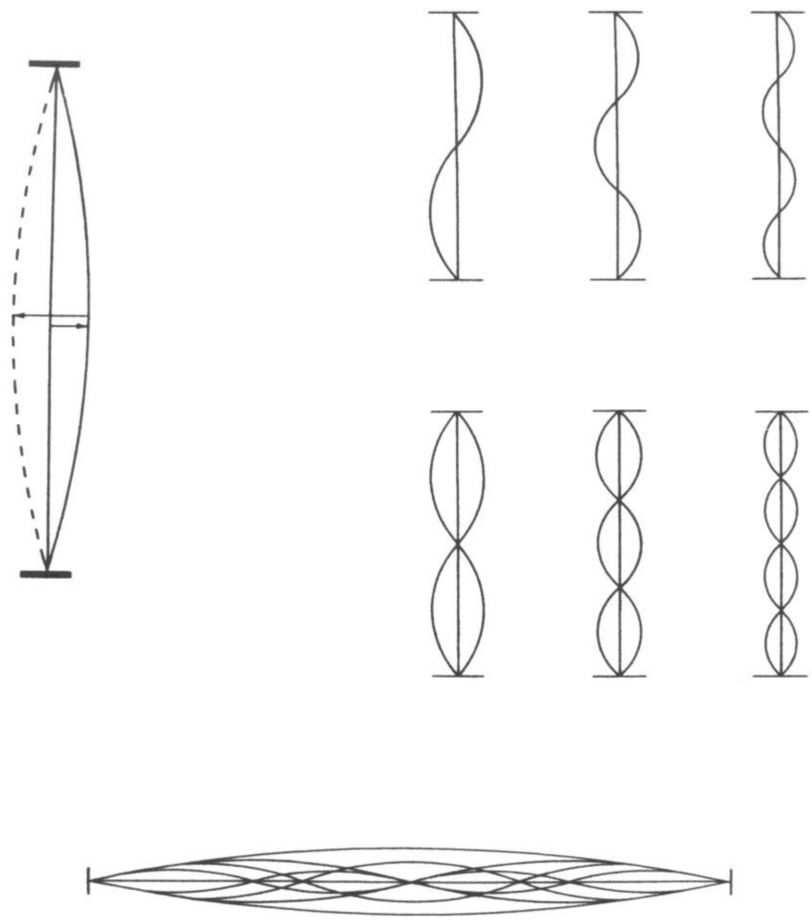


Figure 2-11. The Monochord Wave-Form and the Overtone Wave Series

The Pythagórean theory of musical harmony begins, as does Western music theory in general, with the fact that when any thin elastic chord—a string—is pulled taut and fastened (‘harmózo’ in Greek) at both ends, it can be plucked and made to vibrate. In addition, as we all know, this vibration is audible, and the sound produced is what we now call a tone. Now, in this process, as the string vibrates it moves back and forth *as a whole* at a particular rate or frequency, and this vibrational frequency produces a sound that is now called the *fundamental tone* (the main sound) of a musical scale. The physical vibration of the string itself, however, is not altogether simple, for as the wave-form vibrates along the string, some points on the string are in effect standing still. These stationary of fixed points, called nodes, are located at regular intervals on the string, such as the half-way point, the two thirds, the three fourths, and so on (see the series of six vibrating strings in Figure 2-11). Each of these sections of string, then, vibrates at its own rate, and each section produces a tone of its own (although many of these so-called *overtones* are barely audible). At the same time, each of these vibrating sections is necessarily shorter than the whole string, and since shorter strings mean faster vibrations and thus higher-pitched tones, all of these additional tones are different from the fundamental tone. For example, as the *whole* string vibrates back and forth at the center at a particular *rate*—say, 440 times a second—each *half* of the string vibrates between the half-way point at *twice* the rate—880 times a second—and this secondary vibration produces what is now called the first overtone. A second overtone is created as well, by the vibration of each third of the string moving at three times the frequency of the whole string; and a *third* overtone by each fourth of the string vibrating at four times the frequency of the whole string. Nor is there any end (theoretically) to these overtone vibrations, and as a result the sound of the plucked strings contains not only the fundamental tone of the string, but a potentially infinite collection of these secondary overtones as well. The accumulative result of all these tones is what is now called the *harmonic series of overtones* (illustrated at the bottom of Figure 2-11), and this series of sounds is the basis of all traditional Western music. What Pythagóras discovered as he (purportedly) walked by the blacksmith's shop, then, is the fairly simple mathematical explanation that underlies this series of overtones. For what Pythagóras found as he experimented with the monochord was that, like the weights of the hammers, the lengths of the strings of the lyre or the tubes of a flute were in precisely the ratios of 1-to-2, 2-to-3, and 3-to-4 (as illustrated by the six pairs of strings in the figure). And by applying this seminal intuition to his study of the monochord, Pythagóras and his followers went on to uncover the mathematical nature of the complete harmonic overtone series. From this discovery, using the simplest harmonic ratios, they went on to construct the first complete mathematical theory of an eight tone musical scale, called the Diatonic scale. The first tone (now called the *Key* tone) of the scale is produced by the vibration of the whole length of the string, and the last or eighth tone (now called the octave, from Italian nomenclature), the first overtone, in fact, is created by the vibration of the two halves of the string. In between these two points on the string, the remaining six tones of the eight tone Diatonic scale are determined.

Nowadays we take for granted the notion that the musical scale of eight tones has a mathematical basis, and so this idea does not generate much interest for most people. This familiarity, however, is simply the result of the fact that the Western world has been building upon Pythagóras' original ideas for some twenty-five hundred years now. For the earliest musical theorists, the Pythagóreans, who in comparison had seen nothing like this before, this

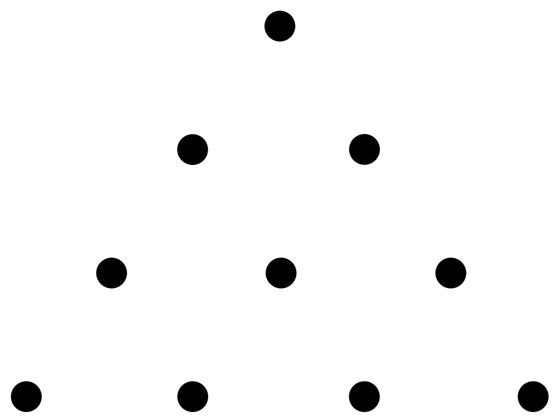


Figure 2-12. Two Forms of the Pythagórean Tetraktys

originary discovery was monumental and seemed divinely inspired; and the significance of their discovery moved them to memorialize the basic musical ratios by weaving them into there most sacred symbol: the symbol of the Tetraktys. This symbol—which is shown at the top of Figure 2-12—is the symbol upon which the members of the Pythagorean brotherhood swore their sacred oaths. And as we see from the diagram, the basic Tetraktys is a set of points representing the triangular number 10. We have noted, however, that the number 10 is in fact the sum of four elements: the two *creative principles*—the Monad and the Dyad—as well as what are for the Greeks the first two numbers—number 3 and number 4. Specifically, that is, $1 + 2 + 3 + 4 = 10$; and taking these four terms in ascending pairs allows us to produce the ratios of 1-to-2, 2-to-3, and 3-to-4, ratios that underlie the Pythagórean tuning. The ratio of the first pair, the ratio 1-to-2, is just the ratio of the whole string to half of the string, the length that produces the *first overtone*; this overtone vibrates as twice the speed of the whole string, which tends to make it sound like the Key tone, only higher. The second overtone, in contrast, produced by a ratio of 2-to-3, sounds distinctly different from the Key. This tone turns out to be the all important fifth tone of the eight-tone scale and is now called the Dominant tone. And finally, the ratio of 3-to-4 is the ratio of the fourth note of this scale, which is now called the Sub-dominant and is only slightly less pivotal than the Dominant. The sacred Tetraktys, thus, involves the most productive

Pythagórean elements (the Monad and the Dyad), embodies the three simplest arithmetic ratios, and thereby symbolizes the musical harmony of the Kosmos.

From this basic Tetraktys the Pythagóreans, being the mathematicians that they were, went on to developed other Tetraktyses, each of which brought out various aspect of their musical theory. In particular, the second Tetraktys, given in Figure 2-12, below, a figure known as the Harmonic Tetraktys represents a fuller (and perhaps the most significant) set of mathematical ratios in Pythagórean tuning. For this Tetraktys, which starts with the number 1 at the top, generates the numbers along the *left* side by *doubling* each time (2, 4, and 8) and generates the

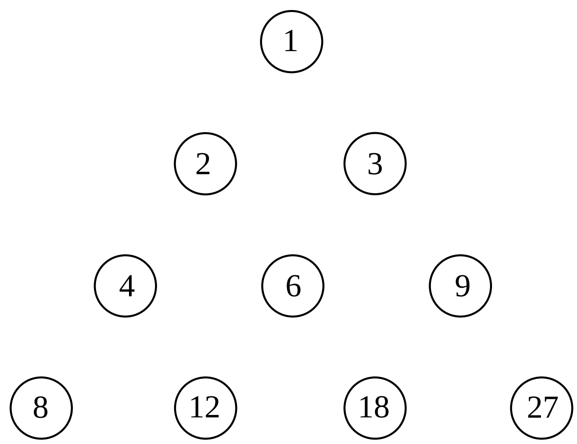


Figure 2-12. Two Forms of the Pythagórean Tetraktys

numbers along the *right* side by *tripling* each time (3, 9, and 27). The numbers in between these two legs of the Tetraktys, then, were generated by multiplying certain numbers located along the legs. For instance, 6, the middle number, is 2 x 3 (the two numbers above 6); 12 is 3 x 4 (as well as 2 x 6); and 18 is 2 x 9 (as well as 3 x 6). And now the primary significance of the Harmonic Tetraktys becomes apparent, for the numbers represent various musical ratios produced from the proper lengths of string. And in fact this Tetraktys embodies a *musical* proportion, for its numbers express the ratios of what the Pythagóreans called the Harmonic Proportion (and that in several different ways). According to the Pythagóreans the Harmonic Proportion begins with two base numbers, 6 and 12, whose ratio is of course 1-to-2 (in this way the mathematicians could avoid using fractions in their musical theory). To these two base numbers Pythagóras added two more numbers, the numbers 8 and 9, which he determined by two different formulas for finding a number in between any two other numbers. The first of these middle numbers or *means* is called the Arithmetic Mean, and it is determined by the formula $A = (n + m)/2$, where A symbolizes the Mean, and m and n are any two numbers. Here, where $n = 6$ and $m = 12$, the Arithmetic Mean, A, turns out to be 9, which numbers is in the Harmonic Tetraktys and has the ratio of 2-to-3 with the lower base, 6, as well as 3-to-4 with the upper base, 12. The first of these ratios is that of the Dominant tone to the Key tone, while the second is the ratio of the Sub-dominant to the Key. For the second mean between 6 and 12, called the Harmonic Mean, the ratios are reversed, in part because this ratio is calculated according to the slightly more complex formula H = (2 x n x m)/(n + m). Using 6 and 12 in this formula, we find that H = 8, and this number, like 9, is in the Harmonic Tetraktys. Unlike 9, however, 8 has the ratio of 3-to-4 with the lower base, 6, and the ratio of 2-to-3 with the upper base, 12.

These new ratios are just the reverse of the ratios determined by A = 9 with 6 and 12, and the Harmonic Proportion thus can take on two forms. The first of these represents the fifth or Dominant note of the Octave, and is written as 6 : 9 :: 8 : 12, in which both ratios are 2-to-3. The second form of the Harmonic Proportion represents the fourth or Sub-dominant note, and is written as 6 : 8 :: 9 : 12. Here, both ratios are 3-to-4, and the full musical meaning of the Harmonic Tetraktys can only be glimpsed by comparing these two Musical Analogies. Finally, and perhaps most significantly, both of these forms of the Harmonic Proportion contain the numbers 8 and 9, and the Pythagóreans used the ratio of these two numbers—whose fractional form is 8/9—to determine the distance between *any* two consecutive notes in the octave or eight-tone Diatonic scale. The Harmonic Proportion, then, and in fact the Harmonic Tetraktys itself, contains all of the mathematical information necessary to build the entire eight-tone musical scale of the Pythagórean tuning system. For this reason alone, the Pythagóreans were justified in feeling that the Tetraktys was so sacred that they swore their sacred oaths on it.

Having seen both Pythagóras' theory of the Kosmos and his theory of Music, we may now consider their significance in terms of the Logic—the relations or linkages—that in particular concern us here. These relations, which are ultimately reducible to natural Contiguity and Isomorphism and (as we have seen in Chapter One), take the form of Metonymy and Analogy for Magic, and take the form of Causality and Analogy for the Science and Philosophy. With respect to the first relation, Causality, we have seen above that Pythagóras' theory of the Kosmos—his theory of the Order that is generated from the Monad and the Dyad—is in effect a *causal* theory of cosmology. And his theory of Music, whose very foundation is Harmonic Proportion, is quite obviously based upon the relation of Analogy, since the Latin term 'proportion' is just a Latin translation of the Pythagórean notion of 'analogos' or Analogy. These two Pythagórean theories thus include both sides of the Logical coin, as it were, with the former focusing on the creative force of Aríthmos and the latter focusing on the organizing form of Harmonía.

References

[1] It should be noted, of course, that this step is a purely mathematical move, there being no notion of ratio without the existence of a system of numbers, such as arithmetic; but this in itself is the same as saying that, given that the only mathematicians of the time were philosophers, this mathematical move was purely philosophical.

[2] The incidents and events in the summary of Pythagoras' life that follows were taken from primary sources in PSL, although PBS and TPP were used as noted.

[3] This particular etymological connection is *especially* poignant, for both 'aríthmos' and 'harmonía' derive from the same root and are thus related in meaning.