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THE BISECTION METHOD FOR SOLVING EQUATIONS

How do computers like your TI calculator solve equations? The really hard way – by guessing and checking. Computers just seem to know the answer right away because they can do their guessing and checking really, really super fast.

In this activity, you will walk through some of the same steps that a computer or calculator goes through to solve an equation. Consider this equation:

$$-6x^3 - x^2 + x + 10 = 0$$

Most equation solving computers need an initial guess. Using your graphing calculator, find two *x*-values between which the solution (aka real zero or *x*-intercept) lies.

The lower *x*-value is commonly called the *left bound* and the higher *x*-value is called the *right bound*. Notice that the *y*-values for the left bound and right bound have different signs. One is negative and the other is positive.

The **Intermediate Value Theorem** guarantees that the solution must lie between the two bounds as long as their *y*-values have different signs.

Your computer is programmed to look somewhere between the two bounds for the solution. Where exactly should the computer look? Normally, the computer splits the difference and takes the midpoint between the bounds as the first best guess for the solution.

This process or *algorithm* for finding solutions is called the *Bisection Method*.

This approach of splitting the difference in half repeats until your calculator gives you the most accurate answer it can. The act of repeating the bisection method is called an *iteration*.

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Sample Table of 2 Iterations for Solving $-6x^3 - x^2 + x + 10 = 0$

Iteration	**Left Bound**	**Midpoint**	**Right Bound**
1) x-value ≈	1.10	**1.15** $= \left(\dfrac{1.10+1.20}{2}\right)$	**1.20**
Is $f(x)$ + or – ?	Positive $f(1.10) \approx 1.9 > 0$	**Positive** $f(1.15) \approx 0.7 > 0$	**Negative** $f(1.20) \approx -0.6 < 0$
2) x-value ≈	**1.15**	$1.175 = \left(\dfrac{1.15+1.20}{2}\right)$	**1.20**
Is $f(x)$ + or – ?	Positive $f(1.15) \approx 0.7 > 0$	Positive $f(1.175) \approx 0.1 > 0$	Negative $f(1.20) \approx -0.6 < 0$

What is the best approximate solution so far after two iterations?

The midpoint at $x = 1.175$ is best approximate solution for the equation so far.

Note 1: Avoid round-off error by not rounding any x-values.
Note 2: Remember to look for the sign change in the y-values to find your next left and right bounds for x.

Starting with the two x-values and midpoint *you* computed above, complete 10 iterations of the bisection method to solve the equation $-6x^3 - x^2 + x + 10 = 0$ in the worksheet below.

Record your work in the boxes of the table following the example from above. After you finish your 10 iterations, what is your best approximation for the zero of the equation?

Record your answer here: ____

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Iteration	Left Bound	Midpoint	Right Bound
1) x-value ≈			
Is $f(x)$ + or – ?			
2) x-value ≈			
Is $f(x)$ + or – ?			
3) x-value ≈			
Is $f(x)$ + or – ?			
4) x-value ≈			
Is $f(x)$ + or – ?			
5) x-value ≈			
Is $f(x)$ + or – ?			
6) x-value ≈			
Is $f(x)$ + or – ?			
7) x-value ≈			
Is $f(x)$ + or – ?			
8) x-value ≈			
Is $f(x)$ + or – ?			
9) x-value ≈			
Is $f(x)$ + or – ?			
10) x-value ≈			
Is $f(x)$ + or – ?			

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