Name: ______________________________ Period: _____ Date: __________________

Tartaglia-Cardano Formula for Cubic Equations

The Rational Zeros Theorem and the Quadratic Formula together can solve most – but not all – cubic polynomial equations. Are you ready to learn the secret to solving more advanced cubic polynomials? ☺

Your mission is to find the exact solutions for this equation from your textbook:

$$-6x^3 - x^2 + x + 10 = 0$$

Observe that your textbook tries to scare you away from finding the solutions by telling you: *do not attempt to solve!* However, you can and will follow the steps below to solve this equation.

Note: please show all necessary arithmetic neatly on separate sheets of paper.

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1. Make the leading coefficient a equal to 1 by dividing each term by -6. Record the resulting equation here:

   ____

2. Next, prepare for the u-substitution. What is the b coefficient? __________
   (The b-coefficient is the coefficient of the quadratic term).

3. Replace each x with $u - \frac{b}{3}$. Use the value for b from the previous step!

4. Simplify and clear all parentheses. Record the resulting equation below:

   ____
   (Hint: the answer should not have a quadratic term).

5. Identify the linear coefficient (called p) and the constant term (called q) in previous result:

   p = _______________ q = _______________

6. Ready for the formula which made Tartaglia and Cardano world famous (at least in 16th century Italy)? Substitute your values for p and q into this short(!!!) formula to obtain the three solutions for the cubic equation:

$$x_1 = \left(\sqrt[3]{\frac{-q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)+\left(\sqrt[3]{\frac{-q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)-\frac{b}{3}$$

$$x_2 = \left(\frac{-1-i\sqrt{3}}{2}\right)\left(\sqrt[3]{\frac{-q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)+\left(\frac{-1+i\sqrt{3}}{2}\right)\left(\sqrt[3]{\frac{-q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)-\frac{b}{3}$$

$$x_3 = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\sqrt[3]{\frac{-q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)+\left(\frac{-1-i\sqrt{3}}{2}\right)\left(\sqrt[3]{\frac{-q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)-\frac{b}{3}$$

Record your result below. Remember to simplify as much as possible without a calculator. In other words, your final answers should include radicals and integers – no decimals!

$x_1 =$ ____

$x_2 =$ ____

$x_3 =$ ____

7. That was easy! After all that work, we want the satisfaction of checking our solutions to make sure that are the actual zeros. To do so, enter each solution into your calculator and record its decimal approximation below (up to and including 9 decimal places):

$x_1 \approx$ ____________________________ checks out? _yes or no_____

$x_2 \approx$ ____________________________ checks out? _yes or no_____

$x_3 \approx$ ____________________________ checks out? _yes or no_____

Plug these approximations into the original equation and verify that the answer is zero (or pretty darn close).