## **Computing the Domain of Composite Functions**

Consider two real-valued functions:

1. 
$$f(x) = \frac{x^2 - x - 6}{x - 3}$$
 and  $g(x) = x - 5$ 

Now compute the composite of these two functions:

2. 
$$f(g(x)) = \frac{(x-5)^2 - (x-5) - 6}{(x-5) - 3} = \frac{x^2 - 11x + 24}{x-8} = \frac{(x-8)(x-3)}{(x-8)}$$
  
3. 
$$f(g(x)) = x - 3$$

What is the domain of the composite?

First analyze the domain of the inner function g(x). The domain of this linear function is all real numbers; there are no restrictions on the domain from g(x).

Next analyze the domain of the outer function f(x). The domain of this rational function is all real numbers except x = 3. To search for any restrictions on the domain of the composite, set the inner function g(x) = 3, the prohibited value for the outer function f(x):

4.  $g(x) = x - 5 = 3 \Longrightarrow x = 8$ 

Thus, the domain of the composite f(g(x)) is  $\{x \mid x \neq 8\}$ .

Observe that if we searched for domain restrictions using *only* the simplified form of the composite in (3), we would have inadvertently skipped over the restriction to the domain.