Finding Zeros

Answer the following questions by creating sample polynomial equations with the specified characteristics.

Solution Methods for Polynomial Equations of Degree Two (Quadratic Polynomials)

1. Take the Square-Root of Both Sides; give a sample equation in which this method would be appropriate: $x^2 = 9$
2. Factor a Trinomial by Grouping; give a sample equation in which this method would be appropriate: ______
3. Factor the Difference of Two Squares; give a sample equation in which this method would be appropriate: ______
4. Use the **Quadratic Formula**; give a sample equation in which this method would be appropriate: ______

Solution Methods for Polynomial Equations of Degree Three (Cubic Polynomials)

5. Factor by Grouping; give a sample equation in which this method would be appropriate: ______
6. Factor the Difference of Two Cubes; give a sample equation in which this method would be appropriate: ______
7. Factor the Sum of Two Cubes; give a sample equation in which this method would be appropriate: ______
8. Use the **Cubic Formula –** only as a last resort!

Solution Methods for Polynomial Equations of Degree Four (Quartic Polynomial)

9. Factor by Grouping; give a sample equation in which this method would be appropriate: ______
10. Factor the Difference of Two Squares; give a sample equation in which this method would be appropriate: ______
11. Factor by Quadratic *Form*; give a sample equation in which this method would be appropriate: ______
12. Use the Quartic Formula – only as a last resort!

General Guidelines for Solving Higher Degree ($n > 2$) Polynomials

The theorems below allow us to determine the number & type of zeros. Knowing the number & type of zeros helps us narrow our search for zeros. **Write a different sample problem to illustrate each of the theorems below.**

1. Fundamental Theorem of Algebra or its corollary on

   The equation $4x^3 + 5x^2 - 3 = 0$ is of degree 3 and has exactly 3 zeros.

2. Odd-Degree Theorem

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3. Descartes Rule of Signs

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4. Intermediate Value Theorem (aka Graphing Theorem)

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5. (Optional) Bounds on Zeros

Next, we use information gained from the theorems above with the techniques listed below to determine the exact value of each zero.

6. Factor if possible and then use the Factor Theorem

   Let $p(x) = x^3 + 2x^2 - x - 2$. By factoring, $p(x) = (x + 2)(x + 1)(x - 1)$. So $p(x)$ has 3 zeros: $x = -2, -1, 1$.

7. List and test possible zeros from Rational Zeros Theorem

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8. Locate and approximate zeros on a graph

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9. Use long division to create a depressed equation & solve the depressed equation to locate additional zeros

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10. Use the Conjugate Roots Theorem

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From Descartes' revolutionary work, *La Geometrie*, (1638) on the discussion of roots of polynomial equations, we find, without hint of a proof, the rule of signs:

> On connoift auffy de cecy combien il peut y auoir de vrayes racines, & combien de fauffes en chafque Equation. A fçauoir il y en peut auoir autant de vrayes, que les fignes + & -- s'y trouuent de fois eftre changés; & autant de fauffes qu'il s'y trouue de fois deux fignes +, ou deux fignes -- quie s'entrefuiuent.
>
> *"We can determine also the number of true and false roots that any equation can have, as follows: An equation can have as many true roots as it contains changes of sign, from + to - or from - to +; and as many false roots as the number of times two + signs or two - signs are found in succession."*

Two Interesting Finding Zeros Problems:

(1) $4x^3 + 4x^2 - 7x + 2 = 0$
Here, Descartes' Sign Rule suggests 2 positive zeros, but the 2 positives are not distinct, that is, x = 0.5 with multiplicity of 2.

(2) $-6x^3 - x^2 + x + 10 = 0$
Here, there is one positive irrational zero. This irrational zero does not have a conjugate pair. Most irrational zeros in your textbook occur as conjugate pairs.

Generalized Conjugate Roots Theorem (not in your textbook)

For a polynomial with rational coefficients and even degree n, the irrational roots, if any, have even multiplicity or occur in conjugate pairs of the form $c \pm d$ where c and/or d are irrational, and the complex roots, if any, occur in conjugate pairs of the form $a \pm bi$.

Example: Determine the roots of the polynomial $x^4 + x^3 - 3x^2 - x + 2$

By the Rational Zeros Theorem, we find three roots x = -2, -1, and 1.
As a consequence of the Fundamental Theorem of Algebra, there must be a fourth root. Is this fourth root a rational, irrational number, or complex number?

By the Generalized Conjugate Roots Theorem (above), complex roots and irrational roots must occur in pairs. Thus, the single remaining root cannot be complex or irrational. The fourth root must be a rational number, that is, one of the three rational roots must have a multiplicity of 2.