

Name: _____

Linear Functions

Please show all the arithmetic or calculator steps you do to arrive at each answer. You may write on the back if needed.

1. Suppose that 650 lb of beans are sold at a price of \$4 per pound, and 400 lb are sold at \$8 per pound.
 - a. List the data points (use price as the independent variable).
 - b. Find the rate of change (slope of the line joining the points).
 - c. Use the point-slope form to write a linear equation for this data. Write the answer in slope-intercept form.
 - d. Use this equation to predict how many pounds of beans consumers would be willing to buy at \$5.60 per pound.
2. A man who is moving into a new apartment plans to rent a truck from a rental company. The cost to rent the truck is \$39.95 for the entire day plus \$0.35 per mile.
 - a. Write a linear equation that expresses the cost of renting the truck based on the number of miles he drives. (Use number of miles as the independent variable.)
 - b. If the distance from the man's old apartment to his new apartment is 12 miles, what will be the cost to drive the truck from his old apartment to his new one and back again?
 - c. If the man has budgeted \$90 for rental expenses, how many miles can he drive the truck and keep his costs within his budget?

Name: _____

Composite Functions

Please show all the arithmetic or calculator steps you do to arrive at each answer. You may write on the back if needed.

3. A size 36 shoe in France is size 3.5 in England. A function that converts shoe sizes in France to those in England is $g(x) = \frac{3x - 94}{4}$. A size 6 shoe in the United States is size 3.5 in England. A function that converts shoe sizes in England to those in the United States is $f(x) = x + \frac{5}{2}$.
- Use composition of functions to find a function that converts shoe sizes in France to those in the United States. Simplify the function.
 - Determine the shoe size in the United States for a size 38 shoe in France.
4. A stone is dropped into a liquid forming circles which increase in radius with time according to the formula $r(t) = 4t$. The area of a circle related to radius is $A(r) = \pi r^2$. Using composition of functions, find an equation that would give area as a function of time.
5. To convert from x degrees Celsius to y degrees Fahrenheit, use the formula $y = f(x) = \frac{9}{5}x + 32$.
- Find the inverse function $g(x)$ that converts from x degrees Fahrenheit to y degrees Celsius.
 - Find $(f \circ g)(x)$ and $(g \circ f)(x)$ to verify that $f(x)$ and $g(x)$ are inverse functions.
6. As the number of units sold increases, market price decreases (supply and demand). Suppose that $p = 5000 - 0.75x$, where p is the market price and x is the number of units sold. Suppose further that the cost of producing x items is given by $C(x) = 3000 + 15x$, and that the revenue from the sale of x units is given by $R(x) = 120x$.
- Express the cost as a function of price, p .
 - Express revenue as a function of price, p .
 - Given that profit is revenue minus cost, express profit as a function of price, p .

Name: _____

Inverse Functions

Cryptology is the science of making and breaking codes. This exercise explores how the idea of functions and their inverses can be used to encode and decode messages.

To encode and decode a message, first replace each letter of the alphabet with a positive integer using the following scheme, thus rewriting the original message as numbers instead of words:

A – 1	F – 6	K – 11	P – 16	U – 21	Z – 26
B – 2	G – 7	L – 12	Q – 17	V – 22	Blank - 27
C – 3	H – 8	M – 13	R – 18	W – 23	
D – 4	I – 9	N – 14	S – 19	X – 24	
E – 5	J – 10	O – 15	T – 20	Y – 25	

ENCODING: A one-to-one function can be used to encode a numerical message. For example, suppose you want to send the message MATH to a friend, and you have decided that the function $f(x) = 3x + 4$ will be the encoding function. This function simply describes the procedure used to create the encoded message – in this case multiply by 3 and add 4. First change the letters to corresponding numbers as shown above: 13 1 20 8. Then use these as the input values in $f(x)$.

$$f(13) = 3(13) + 4 = 43$$

$$f(1) = 7$$

$$f(20) = 64$$

$$f(8) = 28$$

So the encoded message that you send to your friend is: 43 7 64 28

NOTE: A graphing calculator can be used to evaluate a function as above:

- Press [Y=] on the calculator. Type $3x + 4$.
- Choose $[2^{\text{nd}}][\text{WINDOW}]$ (TBLSET). Set the independent variable to Indpnt: Ask.
- Then choose $[2^{\text{nd}}][\text{GRAPH}]$ (TABLE). Input each x value (13 1 20 8). The calculator will return the function values (43 7 64 28) which is the encoded message.

Name: _____

Inverse Functions (continued)

DECODING an encoded message: Now it is up to your friend to decode the message. Decoding is the process that “undoes” the encoding process. If $f(x)$ encodes the message, what will decode it? The inverse of $f(x)$ or $f^{-1}(x)$! So $f^{-1}(x)$ will be the decoding function.

In the example, the inverse of $f(x) = 3x + 4$ can be shown to be $f^{-1}(x) = \frac{x-4}{3}$. Take the encoded message (43 7 64 28) and use these values as input values in $f^{-1}(x)$. Again, the calculator can be used to decode – simply enter $y = \frac{x-4}{3}$ and use the TABLE feature as described above.

$$f^{-1}(43) = 13$$

$$f^{-1}(7) = 1$$

$$f^{-1}(64) = 20$$

$$f^{-1}(28) = 8$$

That's the original numerical message! The last step is to convert back to letters using the table given previously. Now your friend knows the message that you sent: MATH.

Cryptology Applications:

7. Assume that the encoding function is $f(x) = 6x - 2$.

- Find the inverse of $f(x)$.
- Using the inverse function you just found, decode the following encoded message. The answer should be words. Put spaces where appropriate!

16 88 70 70 28 40 28 160 4 70 40 28 10 106 4 160 52 112 160 76

148 160 34 4 130 88 106 52 118 28 160 112 124 10 58 28 16 118

8. Suppose $f(x) = 2x + 7$ is the encoding function.

- Encode “Collin Cougars”. The answer will be numbers.
- Find the inverse of $f(x)$.
- Decode the following. The answer will be words.

47 23 17 61 47 25 33 17 61 25 45 61 35 37 53

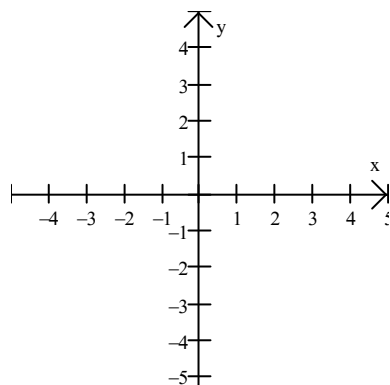
9. Why is it necessary for the encoding function to be one-to-one?

Name: _____

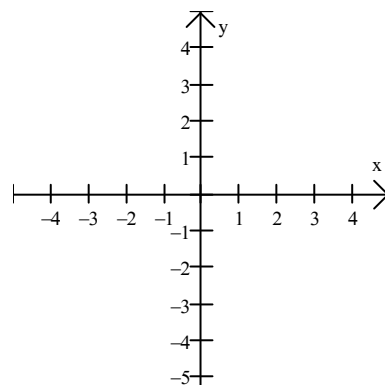
Graphing Piecewise-Defined Functions

10. Graph each function.

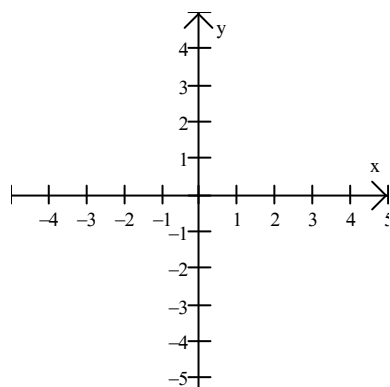
$$f(x) = \begin{cases} -x & \text{if } x < 1 \\ 2 & \text{if } x \geq 1 \end{cases}$$



$$g(x) = \begin{cases} x+2 & \text{if } x \leq -2 \\ -2x-1 & \text{if } x > -2 \end{cases}$$



$$h(x) = \begin{cases} 2x-2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$



$$j(x) = \begin{cases} x+3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$$

