Chapter 2

- Functions and Graphs

2.1

- Basics of Functions & Their Graphs

Objectives

- Find the domain & range of a relation.
- Evaluate a function.
- Graph functions by plotting points.
- Obtain information from a graph.
- Identify the domain & range from a graph.
- Identify *x-y* intercepts from a graph.

Domain & Range

- Domain: first components in the relation (independent variable or *x*-values)
- Range: second components in the relation (dependent variable, the value depends on what the domain value is, aka *y*-values)
- Functions are SPECIAL relations: A domain element corresponds to exactly ONE range element.

EXAMPLE

- Consider the function: eye color
- (Assume all people have only one color)
- It IS a function because when asked the eye color of each person, there is only one answer.
- e.g. {(Joe, brown), (Mo, blue), (Mary, green), (Ava, brown), (Natalie, blue)}
- NOTE: the range values are not necessarily unique.

Evaluating a function

- Common notation: $f(x)$ = function
- Evaluate the function at various values of x, represented as: $f(a)$, $f(b)$, etc.
- Example: $f(x) = 3x – 7$

$$f(2) = 3(2) – 7 = 6 – 7 = -1$$

$$f(3 – x) = 3(3 – x) – 7 = 9 – 3x – 7 = 2 – 3x$$

Graphing a functions

- Horizontal axis: x values
- Vertical axis: y values
- Plot points individually or use a graphing utility (calculator or computer algebra system)
- Example:

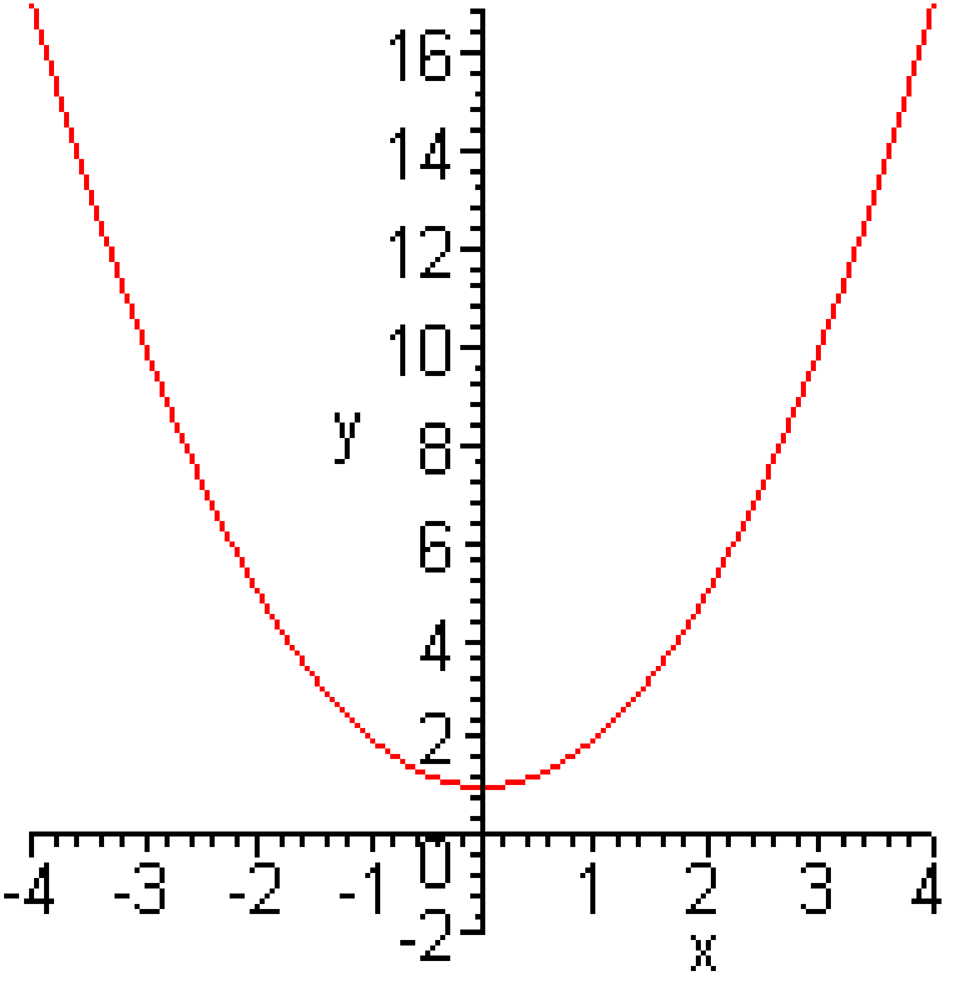
$$y = x^2 + 1$$

Table of function values

$$y = x^2 + 1$$

X (domain)	Y (range)
-4	17
-3	10
-2	5
-1	2
0	1
1	2
2	5
3	10
4	17

Graphs of functions



Can you identify domain & range from the graph?

- Look horizontally. What all x-values are contained in the graph? That’s your domain!
- Look vertically. What all y-values are contained in the graph? That’s your range!

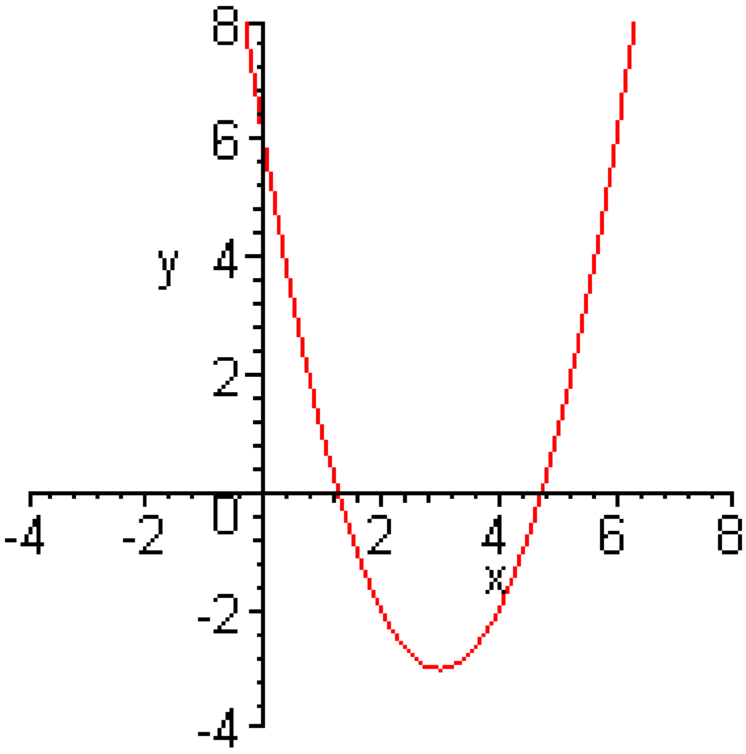
What is the domain & range of the function with this graph?

1) $Domain: (-\infty, \infty), Range: (-\infty, \infty)$

2) $Domain: (-3, \infty), Range: (-\infty, \infty)$

3) $Domain: (-3, \infty), Range: (-3, \infty)$

4) $Domain: (-\infty, \infty), Range: (-3, \infty)$



Correct Answer: 4

Finding intercepts:

- *X-intercept*: where the function crosses the x-axis. What is true of every point on the x-axis? The y-value is ALWAYS zero.
- *Y-intercept*: where the function crosses the y-axis. What is true of every point on the y-axis? The x-value is ALWAYS zero.
- Can the x-intercept and the y-intercept ever be the same point? YES, if the function crosses through the origin!

2.2

- More of Functions and Their Graphs

Objectives

- Find & simplify a function’s difference quotient.
- Understand & use piecewise functions.
- Identify intervals on which a function increases, decreases, or is constant.
- Use graphs to locate relative maxima or minima.
- Identify even or odd functions & recognize the symmetries.

Difference Quotient

- Useful in discussing the rate of change of function over a period of time
- EXTREMELY important in calculus, (h represents the difference in two x values)

$$\frac{f(x+h)-f(x)}{h}$$

Find the difference quotient

$$f(x) = 2x^3 - 2x + 1$$

$$f(x+h) = 2(x+h)^3 - 2(x+h) + 1$$

$$f(x+h) = 2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h + 1$$

$$f(x+h) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x - 2h + 1$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x - 2h + 1 - (2x^3 - 2x + 1)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{6x^2h + 6xh^2 + 2h^3 - 2h}{h} = \frac{h(6x^2 + 6xh \ + 2h^2 - 2)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 6x^2 + 6xh + 2h^2 - 2$$

What is a piecewise function?

- A function that is defined differently for different parts of the domain.
- Examples: You are paid $10/hr for work up to 40 hrs/wk and then time and a half for overtime.

$$f(x)=\begin{cases}10x & \text{if } x\le 40\\ 15x & \text{if } x>40\end{cases}$$

Increasing and Decreasing Functions

- **Increasing**: Graph goes "up" as you move from left to right.

$$x_1 < x_2,\ f(x_1) < f(x_2)$$

- **Decreasing**: Graph goes "down" as you move from left to right.

$$x_1 < x_2,\ f(x_1) > f(x_2)$$

- **Constant**: Graph remains horizontal as you move from left to right.

$$x_1 < x_2,\ f(x_1) = f(x_2)$$

Even & Odd Functions

- Even functions are those that are mirrored through the y-axis. (if –x replaces x, the y value remains the same) (e.g. 1st quadrant reflects into the 2nd quadrant)
- Odd functions are those that are rotated through the origin. (if –x replaces x, the y value becomes –y) (e.g. 1st quadrant reflects into the 3rd quadrant)

Determine if the function is even, odd, or neither.

$$f(x) = 2(x-4)^2 - 2x^2$$

1. Even
2. Odd
3. Neither

Correct Answer: 3

2.3

- Linear Functions & Slope

Objectives

- Calculate a line’s slope.
- Write point-slope form of a line’s equation.
- Model data with linear functions and predict.

What is slope? The steepness of the graph, the rate at which the y values are changing in relation to the changes in x.

How do we calculate it?

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A line has one slope

- Between any 2 pts. on the line, the slope MUST be the same.
- Use this to develop the point-slope form of the equation of the line.

$$y - y_1 = m(x - x_1)$$

- Now, you can develop the equation of any line if you know either a) 2 points on the line or b) one point and the slope.

Find the equation of the line that goes through (2,5) and (-3,4)

1st: Find slope of the line

m= $\frac{5-4}{2-(-3)} = \frac{1}{5}$

2nd: Use either point to find the equation of the line & solve for y.

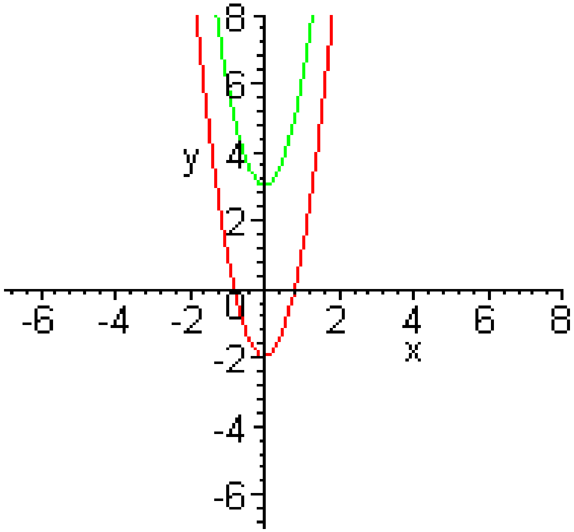
$$y - 5 = \frac{1}{5}(x - 2)$$

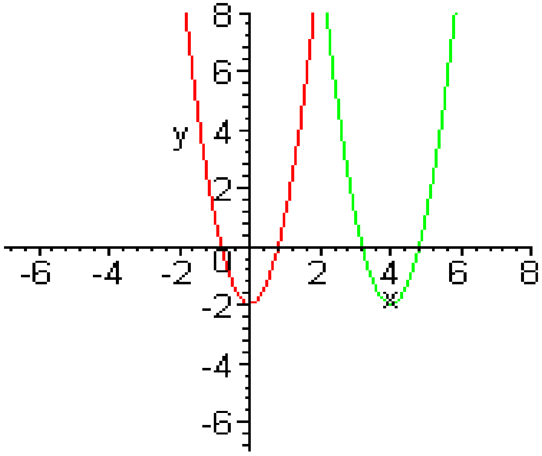
$$y = \frac{1}{5}x - \frac{2}{5} + 5 = \frac{1}{5}x + 4\frac{3}{5}$$

2.5 Transformation of Functions

- Recognize graphs of common functions
- Use vertical shifts to graph functions
- Use horizontal shifts to graph functions
- Use reflections to graph functions
- Graph functions w/ sequence of transformations

- Vertical shifts
  - Moves the graph up or down
  - Impacts only the “y” values of the function
  - No changes are made to the “x” values
- Horizontal shifts
  - Moves the graph left or right
  - Impacts only the “x” values of the function
  - No changes are made to the “y” values





Recognizing the shift from the equation. Examples of shifting the function f(x) =x^2

- Vertical shift of 3 units up

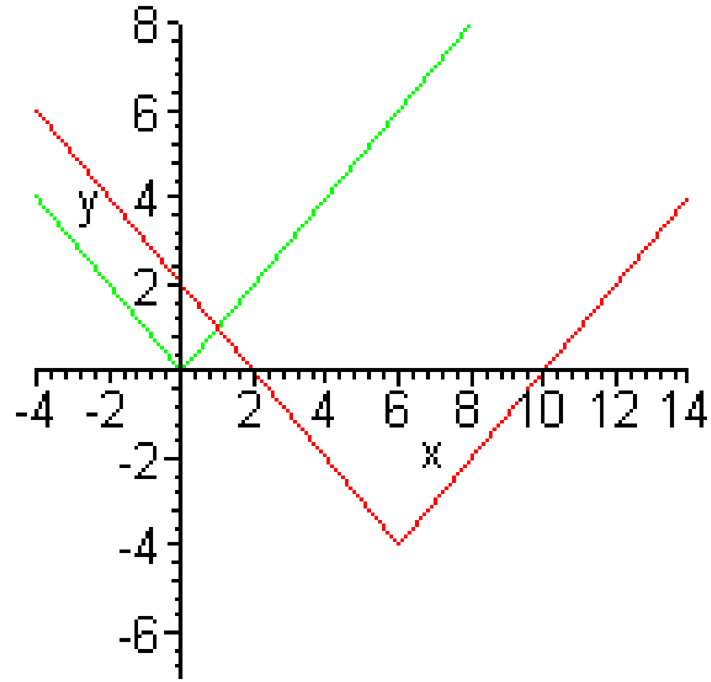
$$f(x) = x^2, h(x) = x^2 + 3$$

- Horizontal shift of 3 units left (HINT: x's go the opposite direction that you might believe.)

$$f(x) = x^2, g(x) = (x+3)^2$$

Combining a vertical & horizontal shift

- Example of function that is shifted down 4 units and right 6 units from the original function.



$$f(x) = |x|, g(x) = |x-6| - 4$$

Reflecting

- Across x-axis (y becomes negative, -f(x))

- Across y-axis (x becomes negative, f(-x))

2.6 Combinations of Functions; Composite Functions

- Objectives
  - Find the domain of a function
  - Form composite functions.
  - Determine domains for composite functions.
  - Write functions as compositions.

Using basic algebraic functions, what limitations are there when working with *real* numbers?

A) You can never divide by zero.

Any values that would result in a zero denominator are NEVER allowed, therefore the domain of the function (possible x values) would be limited.

B) You cannot take the square root (or any even root) of a negative number.

Any values that would result in negatives under an even radical (such as square roots) result in a domain restriction.

Example

- Find the domain $\dfrac{\sqrt{x-2}}{x^2-5x+6}$

- There are x's under an even radical AND x's in the denominator, so we must consider both of these as possible limitations to our domain.

$$x-2 \geq 0,\, x \geq 2$$

$$x^2-5x+6 \neq 0$$

$$(x-3)(x-2) \neq 0,\, x \neq 2,3$$

$$Domain\quad :\{x : x > 2,\, x \neq 3\}$$

Composition of functions

- Composition of functions means the **output** from the inner function becomes the **input** of the outer function.
- f(g(3)) means you evaluate function g at x=3, then plug that value into function f in place of the x.
- Notation for composition:

$$(f \circ g)(x) = f(g(x))$$

2.7 Inverse Functions

- Objectives
  - Verify inverse functions
  - Find the inverse of a function.
  - Use the horizontal line test to deterimine one-to-one.
  - Given a graph, graph the inverse.
  - Find the inverse of a function & graph both functions simultaneously.

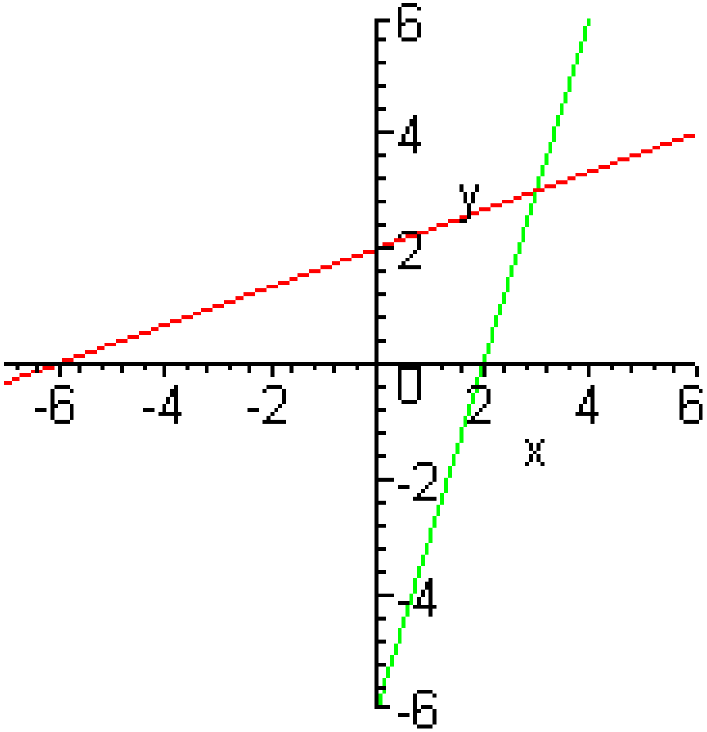
What is an inverse function?

- A function that “undoes” the original function.
- A function “wraps an x” and the inverse would “unwrap the x” resulting in x when the 2 functions are composed on each other.

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

How do their graphs compare?

- The graph of a function and its inverse always mirror each other through the line y=x.
- Example:y = (1/3)x + 2 and its inverse = 3(x-2)
- Every point on the graph (x,y) exists on the inverse as (y,x) (i.e. if (-6,0) is on the graph, (0,-6) is on its inverse.



Do all functions have inverses?

- Yes, and no. Yes, they all will have inverses, BUT we are only interested in the inverses if they ARE A FUNCTION.
- **DO ALL FUNCTIONS HAVE INVERSES THAT ARE FUNCTIONS? NO.**
- Recall, functions must pass the vertical line test when graphed. If the inverse is to pass the vertical line test, the original function must pass the HORIZONTAL line test (be **one-to-one**)!

How do you find an inverse?

- “Undo” the function.
- Replace the x with y and solve for y.