#### CHAPTER 6

#### MATRICES AND DETERMINANTS

# 6.1 Matrix Solutions to Linear Systems

- Objectives
  - Write the augmented matrix for a linear system
  - Perform matrix row operations
  - Use matrices & Gaussian elimination to solve systems
  - Use matrices & Gauss-Jordan elimination to solve systems

### What is a matrix?

- A set of numbers in rows & columns
- m x n describes the dimensions of the matrix (m rows & n columns)
- The matrix is contained within brackets.
- Example of a 3 x 3 matrix:

$$\begin{bmatrix} 4 & 2 & 5 \\ 1 & -4 & 3 \\ 0 & -3 & 8 \end{bmatrix}$$

### Solving a system of 3 equations with 3 variables

- Graphically, you are attempting to find where 3 **planes** intersect.
- If you find 3 numeric values for (x,y,z), this indicates the 3 planes intersect at that point.

### Representing a system of equations in a matrix

- If a linear system of 3 equations involved 3 variables, each column represents the different variables & constant, and each row represents a separate equation.
- Example: Write the following system as a matrix 2x + 3y - 3z = 75x + y - 4z = 2  $\begin{vmatrix} 2 & 3 & -3 & 7 \end{vmatrix}$

$$4x + 2y - z = 6$$
 5 1

### Solving linear systems using Gaussian elimination

- Use techniques learned previously to solve equations (addition & substitution) to solve the system
- Variables are eliminated, but each column represents a different variable
- Perform addition &/or multiplication to simplify rows. Have one row contain 2 zeros, a one, and a constant. This allows you to solve for one variable.
- Work up the matrix and solve for the <sup>4/20/2009</sup> remaining variables.

#### A matrix should look like this after Gaussian elimination is applied

 $\begin{vmatrix} 1 & y_1 & z_1 & k_1 \\ 0 & 1 & z_2 & k_2 \\ 0 & 0 & 1 & k_3 \end{vmatrix}$ 

### Examples 3 & 4 (p. 541-543)

- These examples outline a stepwise approach to use Gaussian elimination.
- One row (often the bottom row) will contain (0 0 0 1 constant) using this method.
- After the last variable is solved in the bottom equation, substitute in for that variable in the remaining equations.

### Question: Must the last row contain (0 0 0 1 constant)?

- Yes, in Gaussian elimination, but there are other options.
- Look again at example 3, page 539.
- If you multiply the first row by (-1) and add it to the 2<sup>nd</sup> row, the result is (-2 0 0 -12)
- What does this mean? -2x = -12, x=6
- By making an informed decision as to what variables to eliminate, we solved for a variable much more quickly!
- Next, multiply row 1 by (-1) & add to 3<sup>rd</sup> row: (-2 2 0 6). Recall x=-6, therefore this becomes
   -2(6)+2y=6, y=3

Now knowing x & y, solve for z in any row (row 2?)

6 + 3 + 2z = 19, z=5 Solution: (6,3,5)

### If this method is quicker, why would we use Gaussian elimination?

- Using a matrix to solve a system by Gaussian elimination provides a standard, programmable approach.
- When computer programs (may be contained in calculators) solve systems. This is the method utilized!

#### 6.2 Inconsistent & Dependent Systems & Their Applications

- Objectives
  - Apply Gaussian elimination to systems without unique solutions
  - Apply Gaussian elimination to systems with more variables than equations
  - Solve problems involving systems without unique solutions.

How would you know, with Gaussian elimination, that there are no solutions to your system?

- When reducing your matrix (attempting to have rows contain only 0's, 1's & the constant) a row becomes 0 0 0 0 k
- What does that mean? Can you have 0 times anything equal to a non-zero constant? NO! No solution!
- Inconsistent system no solution

## Graphically, what is happening with an inconsistent system?

- Recall, with 3 variables, the equation represents a plane, therefore we are considering the intersection of 3 planes.
- If a system is inconsistent, 2 or more of the planes may be parallel OR 2 planes could intersect forming 1 line and a different pair of planes intersect at a different line, therefore there is nothing in common to all three planes.

### Could there be more than one solution?

- Yes! If the planes intersect to form a line, rather than a point, there would be infinitely many solutions. All pts. lying on the line would be solutions.
- You can't state infinitely many points, so you state the general form of all points on the line, in terms of one of the variables.

# What if your system has 3 variables but only 2 equations?

- Graphically, this is the intersection of 2 planes.
- 2 planes cannot intersect in 1 point, rather they intersect in 1 line. (or are parallel, thus no solution)
- The solution is all points on that line.
- The ordered triple is represented as one of the variables (usually z) and the other 2 as functions of that variable: ex: (z+2,3z,z)

#### Dependent system

- Notice when there were infinitely many solutions, two variables were stated in terms of the 3<sup>rd</sup>. In other words, the x & y values are dependent on the value selected for z.
- If there are infinitely many solutions, the system is considered to be **dependent**.

#### 6.5 Determinants & Cramer's Rule

- Objectives
  - Evaluate a 2<sup>nd</sup>-order determinant
  - Solve a system of linear equations in 2 variables using Cramer's rule
  - Evaluate a 3<sup>rd</sup>-order determinant
  - Solve a system of linear equations in 3 variables using Cramer's rule
  - Use determinants to identify inconsistent & dependent systems
  - Evaluate higher-order determinants

#### Determinant of a 2x2 matrix

• If A is a matrix, the determinant is |A|

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}, |A| = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3(5) - (-2)4 = 23$$

#### When are determinants useful?

- They can be used to solve a system of equations
- Cramer's Rule

$$a_{1}x + b_{1}y = c_{1}$$

$$a_{2}x + b_{2}y = c_{2}$$

$$x = \frac{\begin{vmatrix} c_{1} & b_{1} \\ c_{2} & b_{2} \end{vmatrix}}{\begin{vmatrix} a_{1} & b_{1} \\ a_{2} & b_{2} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{1} & c_{1} \\ a_{2} & c_{2} \end{vmatrix}}{\begin{vmatrix} a_{1} & b_{1} \\ a_{2} & b_{2} \end{vmatrix}}$$

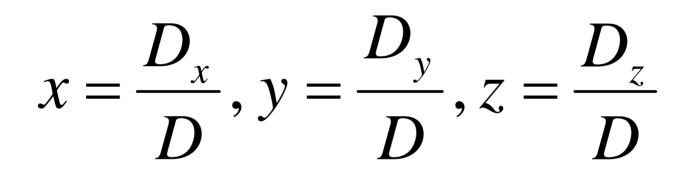
### Finding a determinant of a 3x3 matrix

• More complicated, but it can be done!

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \cdot \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \cdot \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

 It's often easier to pick your "home row/column" (the one with the multipliers) to be a row/column that has one or more zeros in it.

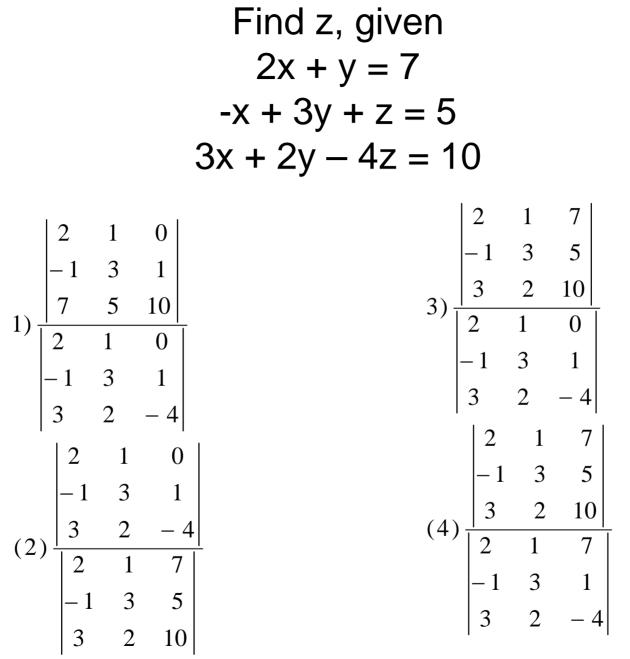
#### Determinants can be used to solve a linear system in 3 variables CRAMER'S RULE



### What is D and $D_x, D_y, D_z$

- D is the determinant that results from the coefficients of all variables.
- $D_x$  is the determinant that results when each x coefficient is replaced with the given constants.
- $D_y$  is the determinant that results when each y coefficient is replaced with the given constants.
- • $D_z$  is the determinant that results when the z coefficients are replaced with the given constants.





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