Chapter 1 – Section 1 Solving Linear Equations in One Variable

A linear equation in one variable is an equation which can be written in the form: ax+b=cfor a, b, and c real numbers with $a \neq 0$. Linear equations in one variable: 2x + 3 = 112(x-1) = 8 can be rewritten 2x + (-2) = 8. $\frac{2}{3}x + 5 = x - 7$ can be rewritten $-\frac{1}{3}x + 5 = -7$. Not linear equations in one variable: $\frac{2}{3x} + 5 = x - 7$ 2x + 3y = 11 $(x-1)^2 = 8$ Two variables x is squared. Variable in the denominator A **solution** of a linear equation in one variable is a real number which, when substituted for the variable in the equation, makes the equation true.

Example: Is 3 a solution of 2x + 3 = 11?

2x + 3 = 11 Original equation 2(3) + 3 = 11 Substitute 3 for x. $6 + 3 \not\asymp 11$ False equation 3 is not a solution of 2x + 3 = 11.

Example: Is 4 a solution of 2x + 3 = 11? 2x + 3 = 11 Original equation 2(4) + 3 = 11 Substitute 4 for x. 8 + 3 = 11 True equation 4 is a solution of 2x + 3 = 11.

Addition Property of Equations

If a = b, then a + c = b + c and a - c = b - c.

That is, the same number can be added to or subtracted from each side of an equation without changing the solution of the equation.

Use these properties to solve linear equations.

Example: Solve x - 5 = 12.

x - 5 = 12x - 5 + 5 = 12 + 5x = 1717 - 5 = 12

The solution is preserved when 5 is added to both sides of the equation. 17 is the solution.

Check the answer.

Original equation

Multiplication Property of Equations If a = b and $c \neq 0$, then ac = bc and $\frac{a}{c} = \frac{b}{c}$.

That is, an equation can be multiplied or divided by the same nonzero real number without changing the solution of the equation.

Example: Solve 2x + 7 = 19.

2x + 7 = 19 2x + 7 - 7 = 19 - 7 2x = 12 $\frac{1}{2}(2x) = \frac{1}{2}(12)$ x = 6Original equation The solution is presubtracted from box Simplify both side The solution is preis multiplied by $\frac{1}{2}$ 6 is the solution. 2(6) + 7 = 12 + 7 = 19Check the answer.

Original equation The solution is preserved when 7 is subtracted from both sides. Simplify both sides. The solution is preserved when each side is multiplied by $\frac{1}{2}$. 6 is the solution. 19 Check the answer.

To solve a linear equation in one variable:

- 1. Simplify both sides of the equation.
- 2. Use the addition and subtraction properties to get all variable terms on the left-hand side and all constant terms on the right-hand side.
- 3. Simplify both sides of the equation.
- 4. Divide both sides of the equation by the coefficient of the variable.

Example: Solve x + 1 = 3(x - 5).

x + 1 = 3(x - 5)Original equationx + 1 = 3x - 15Simplify right-hand side.x = 3x - 16Subtract 1 from both sides.-2x = -16Subtract 3x from both sides.x = 8Divide both sides by -2.

The solution is 8. Check the solution: $(8) + 1 = 3((8) - 5) \rightarrow 9 = 3(3)$ True **Example**: Solve 3(x + 5) + 4 = 1 - 2(x + 6).

3(x+5) + 4 = 1 - 2(x+6)	Original equation
3x + 15 + 4 = 1 - 2x - 12	Simplify.
3x + 19 = -2x - 11	Simplify.
3x = -2x - 30	Subtract 19.
5x = -30	Add 2 <i>x</i> .
x = -6	Divide by 5.
The solution is -6 .	
3(-6+5)+4 = 1 - 2(-6+6)	Check.
3(-1) + 4 = 1 - 2(0)	
-3 + 4 = 1	True

Equations with fractions can be simplified by multiplying both sides by a common denominator.

Example: Solve	$\frac{1}{2}x +$	$\frac{2}{3} = \frac{1}{3}(x+4).$	The lowest common denominator of all fractions in the equation is 6.
$6\left(\frac{1}{2}\right)$	$\frac{2}{x} + \frac{2}{3}$	$- = 6 \left(\frac{1}{3} (x+4) \right)$	Multiply by 6.
	3x + 4	4 = 2x + 8	Simplify.
	3.	x = 2x + 4	Subtract 4.
		x = 4	Subtract 2 <i>x</i> .
$\frac{1}{2}$	- (4) + -	$\frac{2}{3} = \frac{1}{3}((4) + 4)$	Check.
	2+-	$\frac{2}{3} = \frac{1}{3}(8)$	
		$\frac{8}{3} = \frac{8}{3}$	True

Alice has a coin purse containing \$5.40 in dimes and quarters. There are 24 coins all together. How many dimes are in the coin purse?

Let the number of dimes in the coin purse = d. Then the number of quarters = 24 - d.

 \rightarrow 10d + 25(24 - d) = 540

10d + 600 - 25d = 540

10d - 25d = -60

Linear equation Simplify left-hand side. Subtract 600.

-15d = -60

d = 4

Simplify right-hand side.

Divide by –15.

There are 4 dimes in Alice's coin purse.

The sum of three consecutive integers is 54. What are the three integers?

Three *consecutive integers* can be represented as n, n + 1, n + 2.

→ n + (n + 1) + (n + 2) = 54

3n + 3 = 54Simplify left-hand side.

Linear equation

3n = 51Subtract 3.

n = 17Divide by 3.

The three consecutive integers are 17, 18, and 19.

17 + 18 + 19 = 54. Check.



Digital Lesson

Linear Equations in Two Variables Equations of the form ax + by = c are called **linear equations in two variables**.

This is the graph of the equation 2x + 3y = 12.



The point (0,4) is the *y*-intercept. The point (6,0) is the *x*-intercept.

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The **slope** of a line is a number, *m*, which measures its steepness.



The **slope** of the line passing through the two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \ (x_1 \neq x_2).$$

The slope is the change in y divided by the change in x as we move along the line from (x_1, y_1) to (x_2, y_2) .



Example: Find the slope of the line passing through the points (2, 3) and (4, 5).

Use the slope formula with $x_1 = 2$, $y_1 = 3$, $x_2 = 4$, and $y_2 = 5$.



A linear equation written in the form y = mx + b is in **slope-intercept form**.

The slope is m and the y-intercept is (0, b).

To graph an equation in *slope-intercept form*:

1. Write the equation in the form y = mx + b. Identify *m* and *b*.

2. Plot the y-intercept (0, b).

3. Starting at the *y*-intercept, find another point on the line using the slope.

4. Draw the line through (0, *b*) and the point located using the slope.

Example: Graph the line y = 2x - 4.

1. The equation y = 2x - 4 is in the slope-intercept form. So, m = 2 and b = -4.

(0, -4)

2. Plot the y-intercept, (0, -4).

- 3. The slope is 2. $m = \frac{\text{change in } y}{\text{change in } x} = \frac{2}{1}$
- 4. Start at the point (0, 4).Count 1 unit to the right and 2 units up to locate a second point on the line.The point (1, -2) is also on the line.
- 5. Draw the line through (0, 4) and (1, -2).

A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**.

The graph of this equation is a line with slope m passing through the point $(x_1, y_1)_{.}$

Example: The graph of the equation $y-3 = -\frac{1}{2}(x-4)$ is a line of slope $m = -\frac{1}{2}$ passing through the point (4, 3).



Example: Write the slope-intercept form for the equation of the line through the point (-2, 5) with a slope of 3.

Use the point-slope form, $y - y_1 = m(x - x_1)$, with m = 3 and $(x_1, y_1) = (-2, 5)$.

 $y - y_1 = m(x - x_1)$ Point-slope form

 $y - y_1 = 3(x - x_1)$ Let m = 3.

 y - 5 = 3(x - (-2)) Let $(x_1, y_1) = (-2, 5)$.

 y - 5 = 3(x + 2) Simplify.

 y = 3x + 11 Slope-intercept form

Example: Write the slope-intercept form for the equation of the line through the points (4, 3) and (-2, 5).

$$m = \frac{5-3}{-2-4} = -\frac{2}{6} = -\frac{1}{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{1}{3}(x - 4)$$
$$y = -\frac{1}{3}x + \frac{13}{3}$$

Calculate the slope.

Point-slope form

Use
$$m = -\frac{1}{3}$$
 and the point (4, 3).

Slope-intercept form

Two lines are **parallel** if they have the same slope. If the lines have slopes m_1 and m_2 , then the lines are parallel whenever $m_1 = m_2$.

(0,

(0, -3)

y = 2x + 4

Example: The lines y = 2x - 3and y = 2x + 4 have slopes $m_1 = 2$ and $m_2 = 2$.

The lines are parallel.

y = 2x - 3

Two lines are **perpendicular** if their slopes are negative reciprocals of each other. If two lines have slopes m_1 and m_2 , then the lines are perpendicular whenever

(0,

(0, -1)

$$m_2 = -\frac{1}{m_1}$$
 or $m_1 m_2 = -1$

Example:

The lines y = 3x - 1 and $y = -\frac{1}{3}x + 4$ have slopes $m_1 = 3$ and $m_2 = -\frac{1}{3}$.

The lines are perpendicular.

= 3x - 1