Using and Understanding

Mathematics

A Quantitative Reasoning Approach

Third Edition

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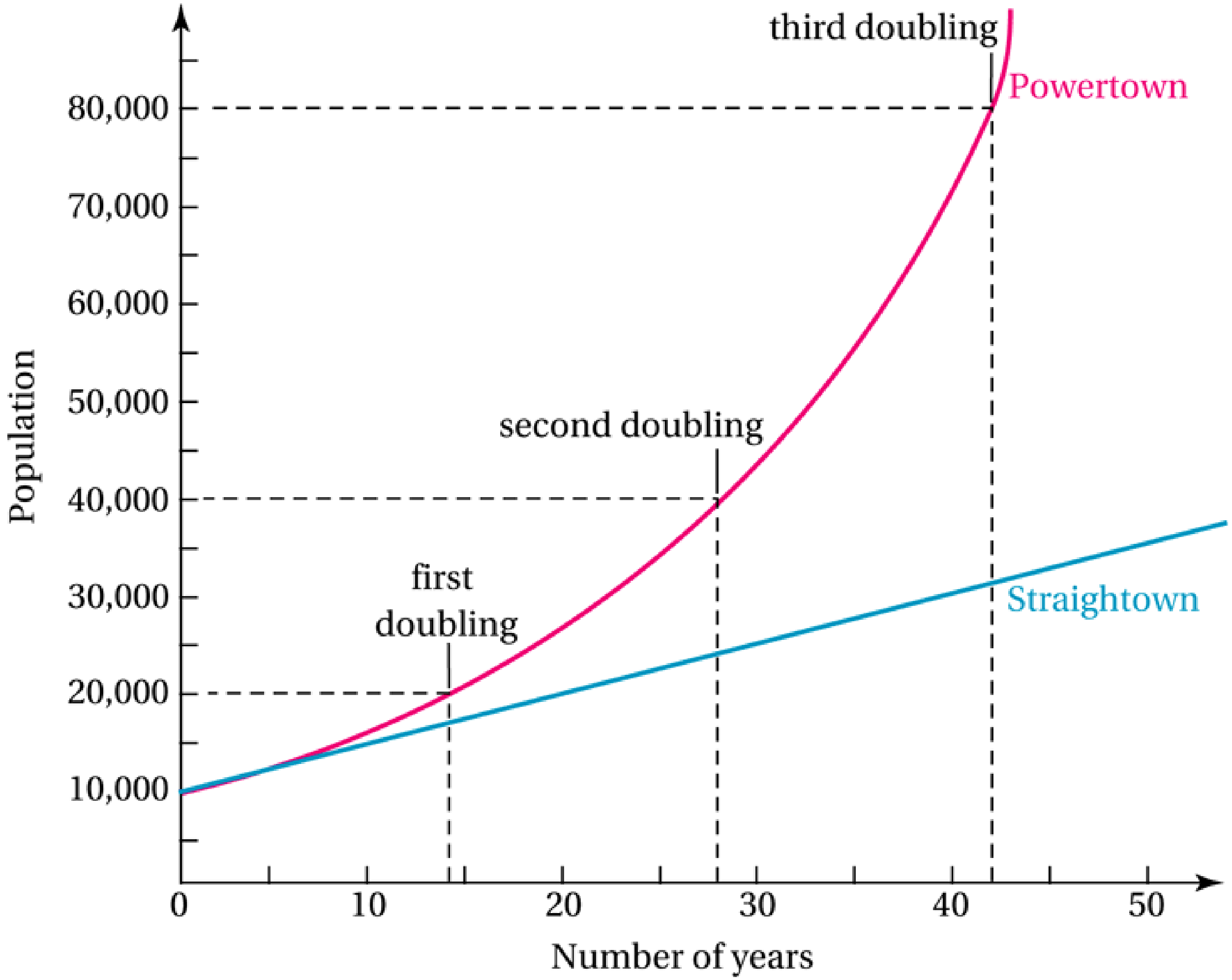
Chapter 8

Growth: Linear versus Exponential



Growth: Linear versus Exponential

- **Linear Growth** occurs when a quantity grows by some fixed *absolute* amount in each unit of time.

- **Exponential Growth** occurs when a quantity grows by the same fixed *relative* amount—that is, by the same percentage—in each unit of time.

Key Facts about Exponential Growth

- **Exponential growth** leads to repeated doublings. With each doubling, the amount of increase is approximately equal to the sum of all preceding doublings.

- **Exponential growth** cannot continue indefinitely. After only a relatively small number of doublings, exponentially growing quantities reach impossible proportions.

Rule of 70 Approximations

- **Doubling Time**

For a quantity growing exponentially at a rate of P% per time period, the doubling time is approximately

$$\boxed{T_{\text{double}} \approx \frac{70}{P}}$$

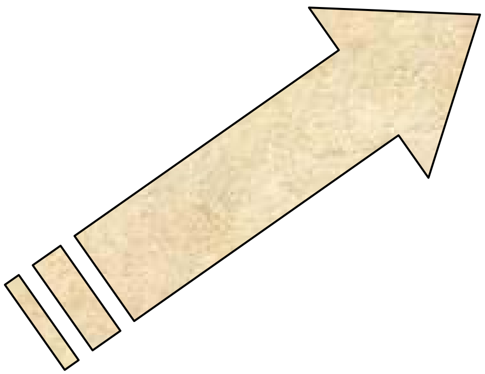
- **Half-Life Period**

For a quantity growing exponentially at a rate of P% per time period, the doubling time is approximately

$$\boxed{T_{\text{half-life}} \approx \frac{70}{P}}$$

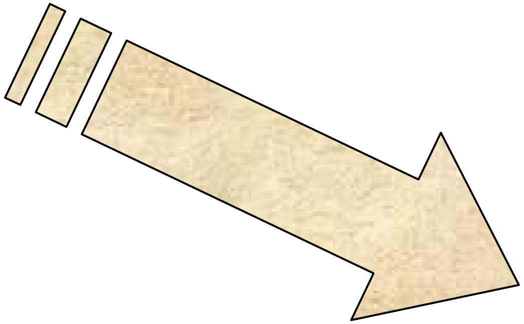
For both doubling and halving, this approximation works best for small growth rates and breaks down for growth rates over about 15%

Exponential Growth vs. Decay



Exponential Growth

$$\text{new value} = \text{initial value} \ \times \ 2^{t/T_{double}}$$



$$\text{new value} = \text{initial value} \ \times \ \left(\frac{1}{2}\right)^{t/T_{half-life}}$$

Exponential Decay

Exact Doubling Time and Half-Life Formulas

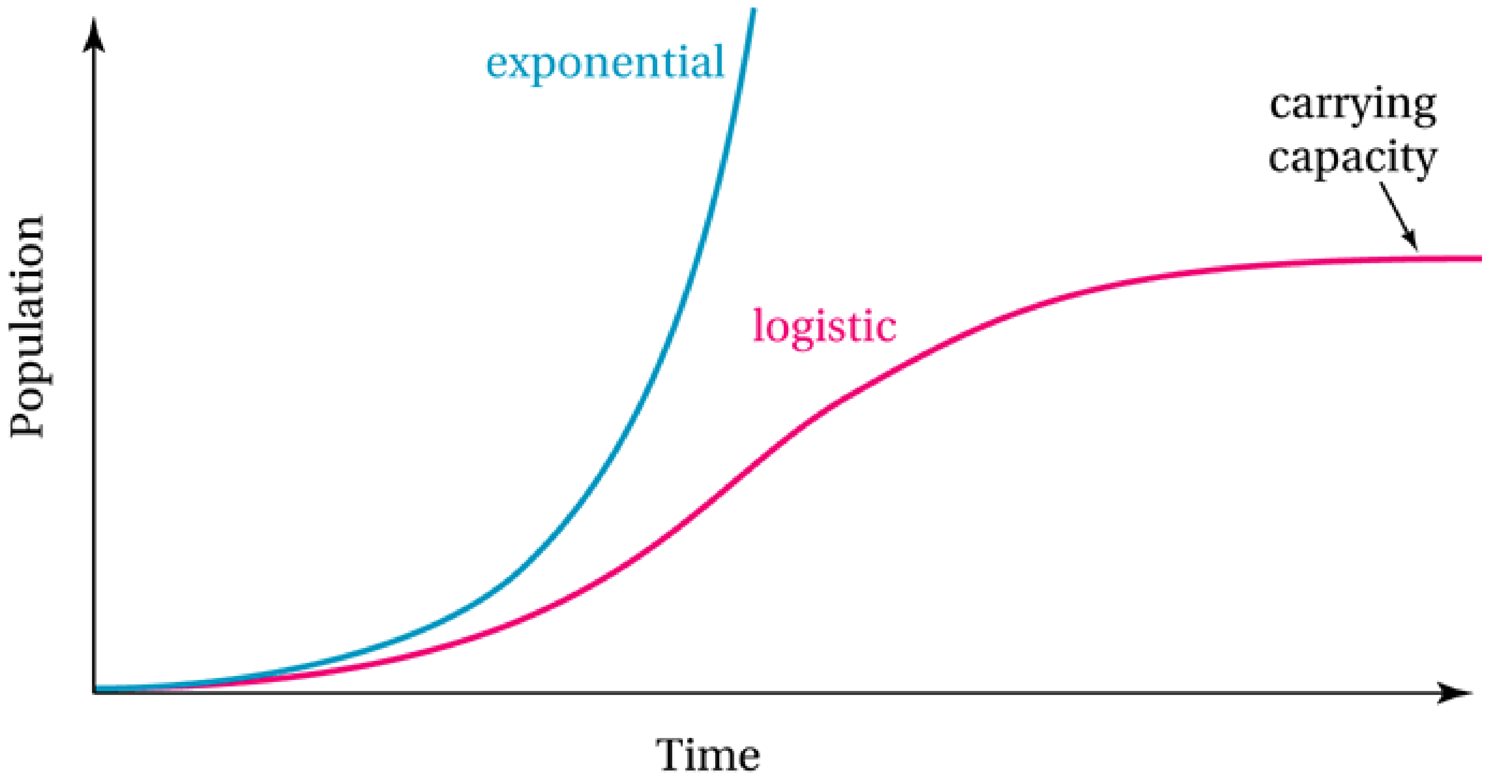
$$T_{\text{double}} = \frac{\log 2}{\log(1+r)}$$

Exponential Growth
(given that $r > 0$)

$$T_{\text{half}} = \frac{-\log 2}{\log(1+r)} \text{ or } \frac{\log(1/2)}{\log(1+r)}$$

Exponential Decay
(given that $r < 0$)

Exponential versus Logistic Growth



Overall Growth Rate and Logistic Growth Rate

Overall Growth

The world population growth rate is the difference between the birth rate and the death rate:

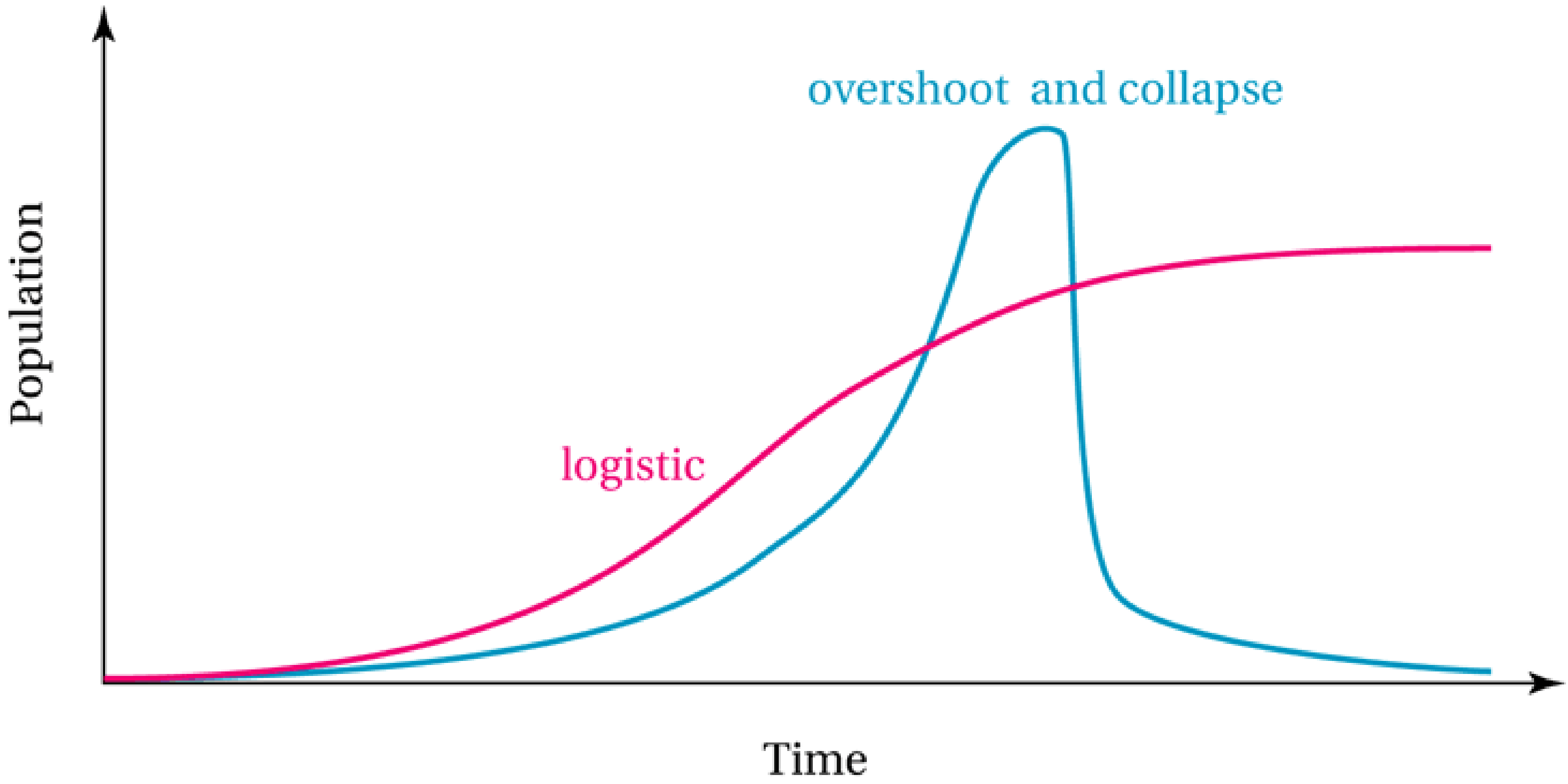
growth rate = birth rate – death rate

Logistic Growth

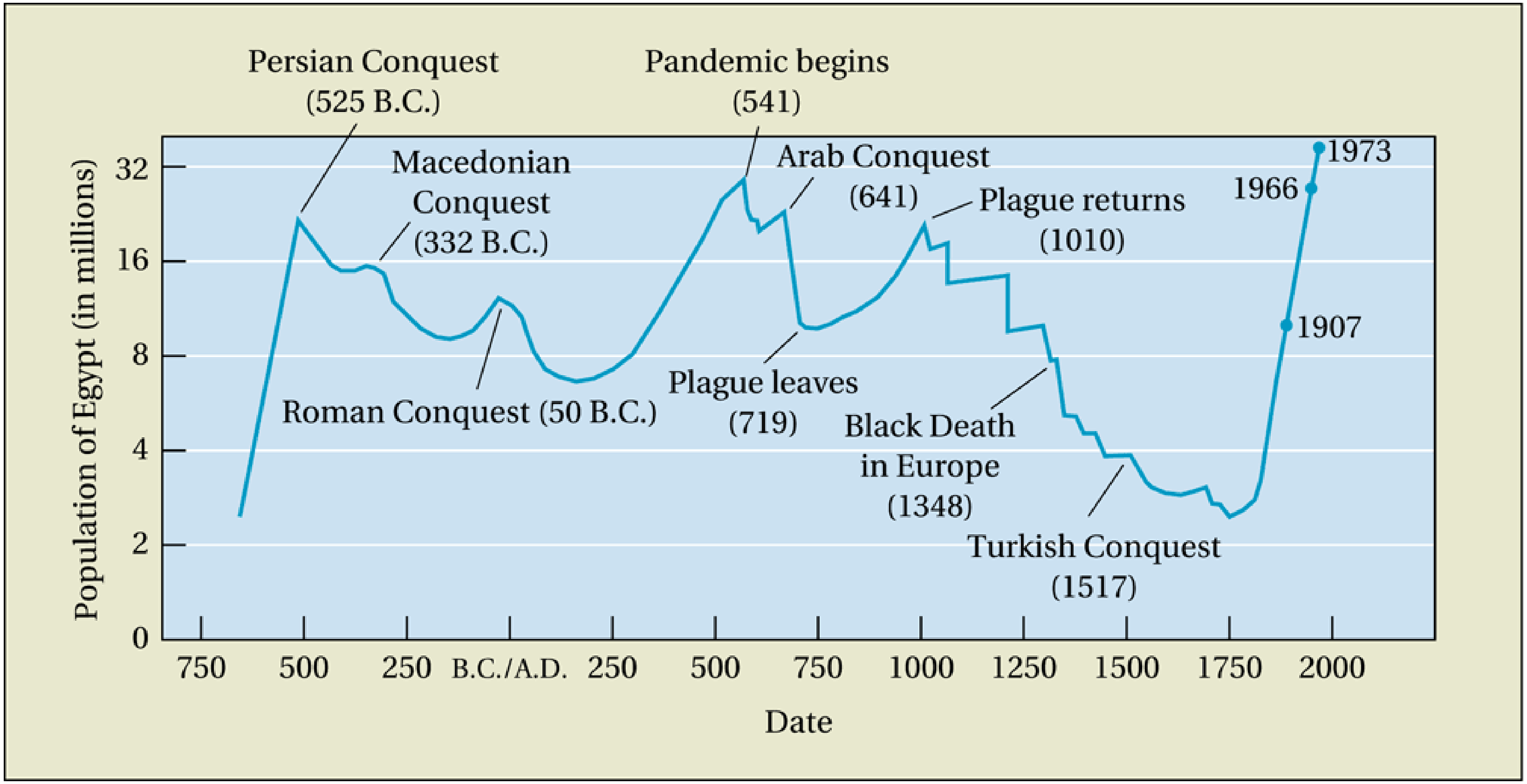
The logistic growth rate is dependent upon the ratio of current population to carrying capacity:

growth rate =
base rate x (1 – population / carrying capacity)

Overshoot and Collapse



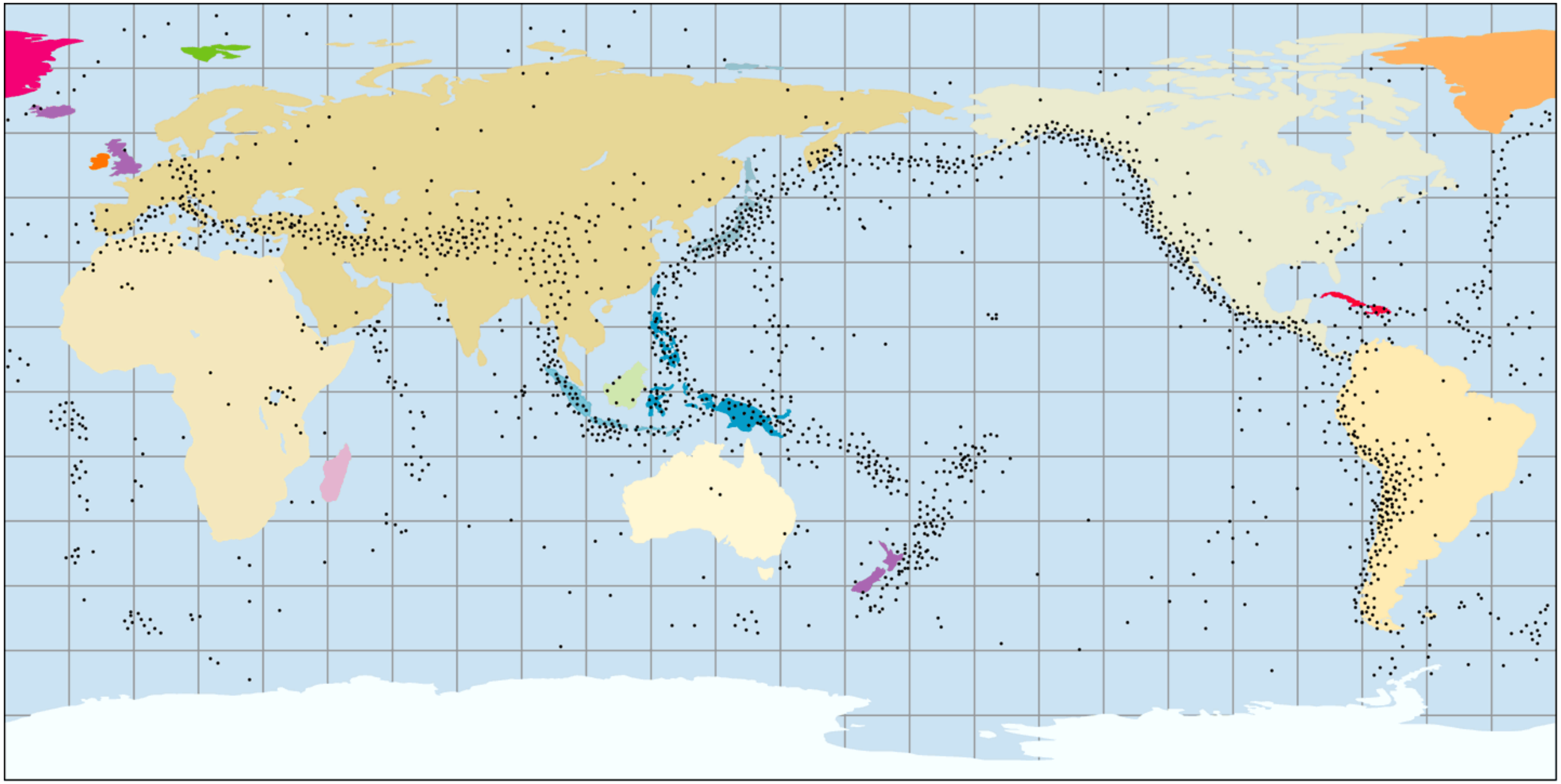
Difficulties of Population Prediction



The historical population of Egypt.

SOURCE: T.H. Hollingsworth, *Historical Demography* (Ithaca, NY: Cornell University Press, 1969).

Distribution of Earthquakes



The distribution of earthquakes around the world.
Each dot represents an earthquake.
SOURCE: U.S. Geological Survey

Measuring Sound

- The **decibel scale** is used to compare the loudness of sounds.
- The loudness of a sound in decibels is defined by the following equivalent formulas:

$$\text{loundness in dB} = 10\log_{10}\left(\frac{\text{intensity of the sound}}{\text{intensity of the softest audible count}}\right)$$

or

$$\frac{\text{intensity of the sound}}{\text{intensity of the softest audible sound}} = 10^{\frac{\text{loudenss in dB}}{10}}$$

Typical Sounds in Decibels

TABLE 8.5 **Typical Sounds in Decibels**

DECIBELS	TIMES LOUDER THAN SOFTEST AUDIBLE SOUND	EXAMPLE
140	10^{14}	jet at 30 meters
120	10^{12}	strong risk of damage to human ear
100	10^{10}	siren at 30 meters
90	10^{9}	threshold of pain for human ear
80	10^{8}	busy street traffic
60	10^{6}	ordinary conversation
40	10^{4}	background noise in average home
20	10^{2}	whisper
10	10^{1}	rustle of leaves
0	1	threshold of human hearing
-10	0.1	inaudible sound

pH Scale

- The pH scale is defined by the following equivalent formula:

  $pH = -\log_{10}[H^+]$ or $[H^+] = 10^{-pH}$

- where $[H^+]$ is the hydrogen ion concentration in moles per liter. Pure water is neutral and has a pH of 7. Acids have a pH lower than 7 and bases have a pH higher than 7.