Using and Understanding

Mathematics

A Quantitative Reasoning Approach

Third Edition

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Chapter 6

Measures of Center in a Distribution

The **mean** is what we most commonly call the average value. It is defined as follows:

$$\text{mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

The **median** is the middle value in the sorted data set (or halfway between the two middle values if the number of values is even).

The **mode** is the most common value (or group of values) in a distribution.

Middle Value for a Median

6.72	3.46	3.60	6.44	26.70	
3.46	3.60	6.44	6.72	26.70	**(sorted list)**

(odd number of values)

exact middle

median is 6.44

No Middle Value for a Median

6.72 3.46 3.60 6.44

3.46 3.60 6.44 6.72 **(sorted list)**

(even number of values)

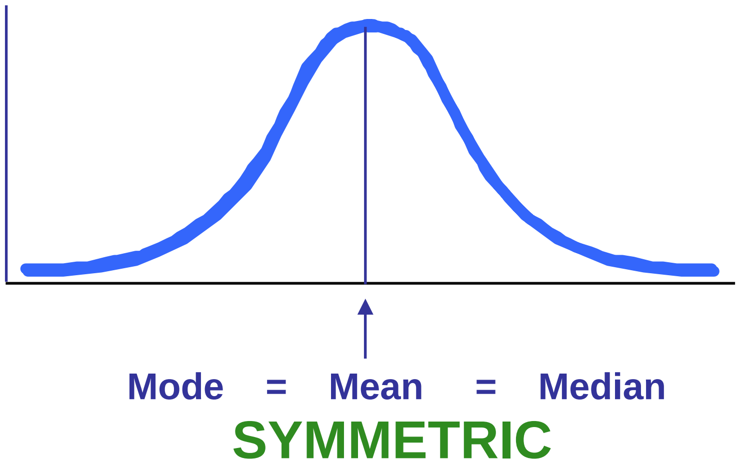
$$\frac{3.60 + 6.44}{2}$$

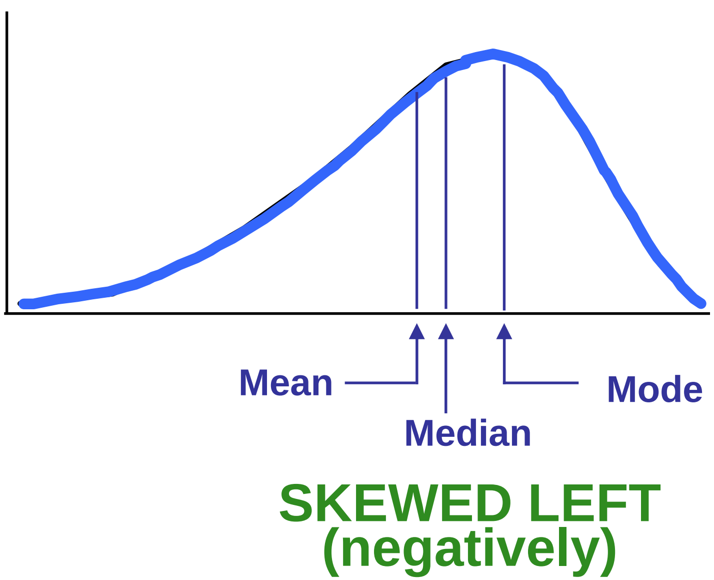
median is 5.02

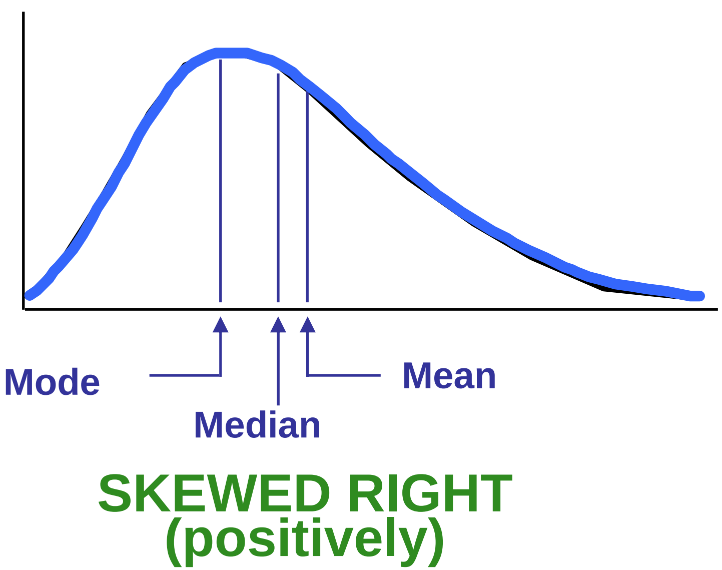
Mode Examples

a. 5 5 5 3 1 5 1 4 3 5	Mode is 5
b. 1 2 2 2 3 4 5 6 6 6 7 9	Bimodal (2 and 6)
c. 1 2 3 6 7 8 9 10	No Mode

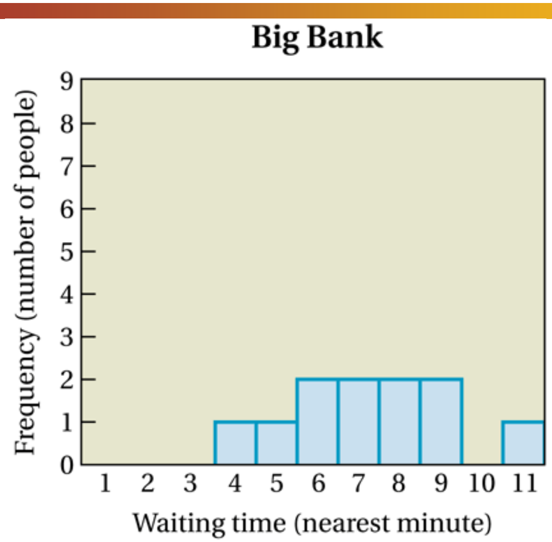
Symmetric and Skewed Distributions





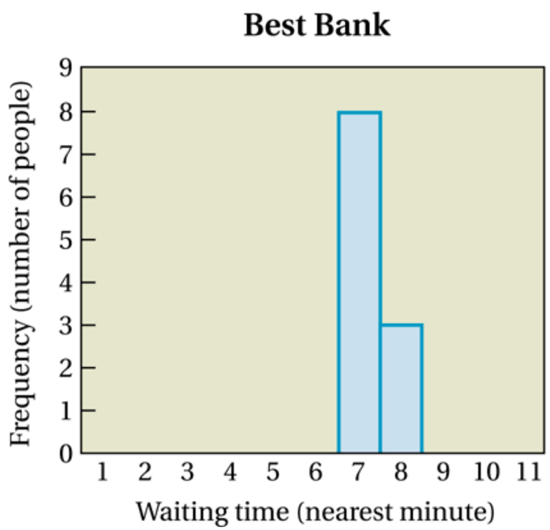


Why Variation Matters



Big Bank (three line wait times):

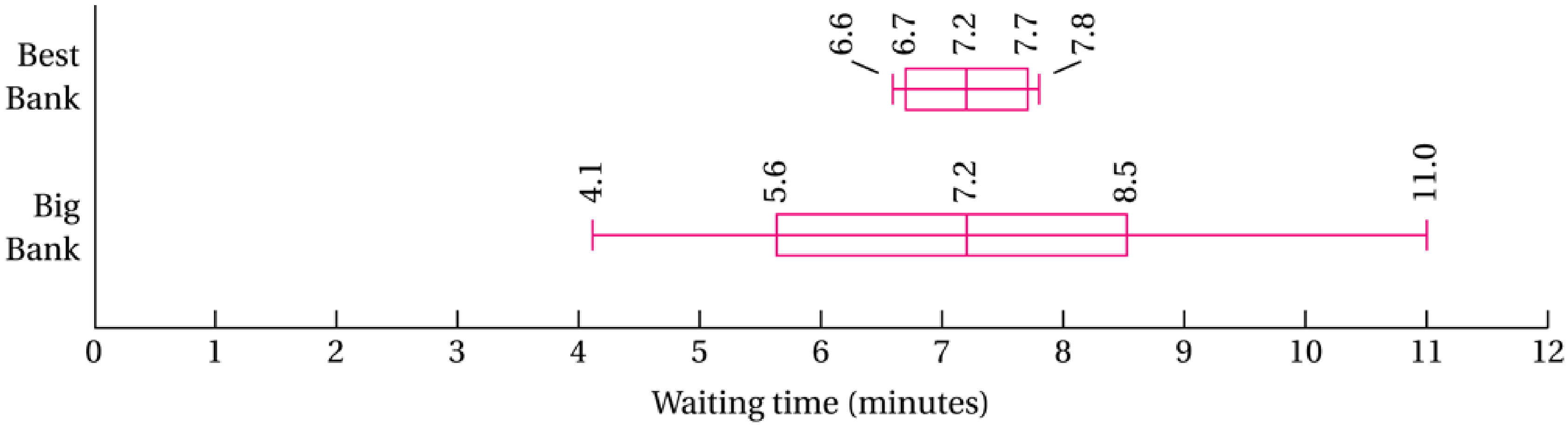
4.1	5.2	5.6	6.2	6.7	7.2
7.7	7.7	8.5	9.3	11.0	



Best Bank (one line wait times):

6.6	6.7	6.7	6.9	7.1	7.2
7.3	7.4	7.7	7.8	7.8	

Five Number Summaries & Box Plots



Big Bank		
low value (min)	=	4.1
lower quartile	=	5.6
median	=	7.2
upper quartile	=	8.5
high value (max)	=	11.0

Best Bank		
low value (min)	=	6.6
lower quartile	=	6.7
median	=	7.2
upper quartile	=	7.7
high value (max)	=	7.8

Standard Deviation

Let A = {2, 8, 9, 12, 19} with a mean of 10. Use the data set A above to find the sample standard deviation.

x (data value)	x – mean (deviation)	$(\text{deviation})^2$
2	2 – 10 = -8	$(-8)^2 = 64$
8	8 – 10 = -2	$(-2)^2 = 4$
9	9 – 10 = -1	$(-1)^2 = 1$
12	12 – 10 = 2	$(2)^2 = 4$
19	19 – 10 = 9	$(9)^2 = 81$
	Total	**154**

$$\text{standard deviation} = \sqrt{\frac{154}{5-1}} = 6.2$$

$$\text{standard deviaton} = \sqrt{\frac{\text{sum of (deviations from the mean)}^2}{\text{total number of data values} - 1}}$$

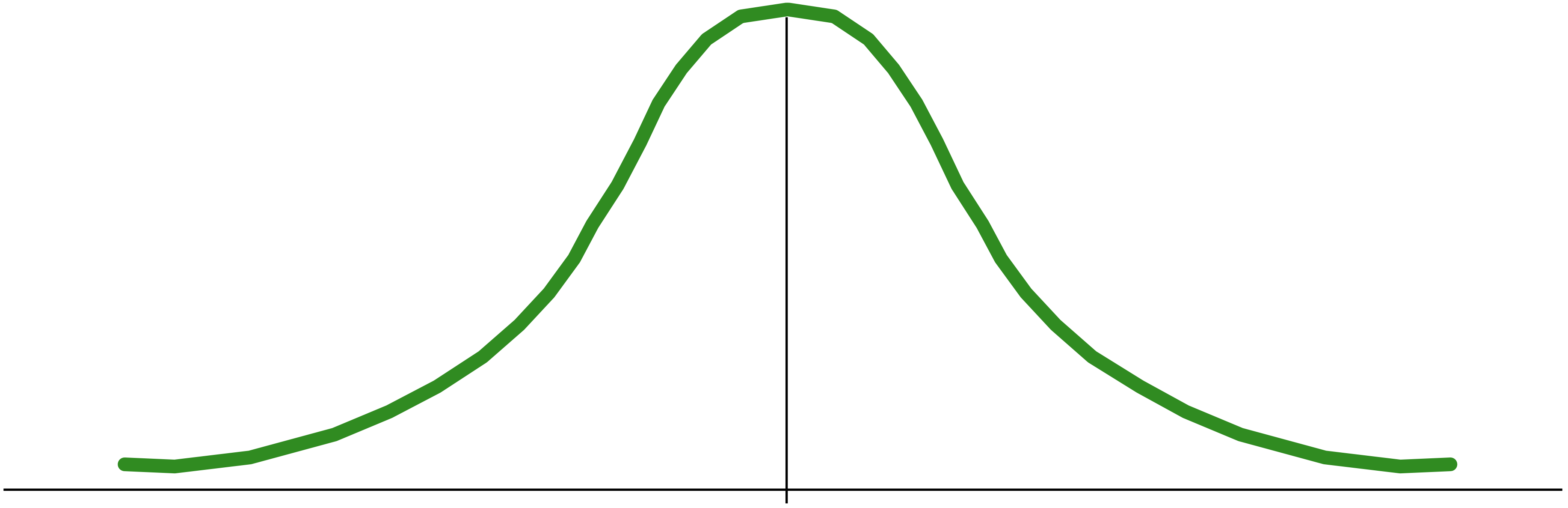
Range Rule of Thumb for Standard Deviation

$$\text{standard deviation} \approx \frac{\text{range}}{4}$$

We can estimate standard deviation by taking an approximate range,
(usual high – usual low), and dividing by 4. The reason for the 4 is due to the idea that the usual high is approximately 2 standard deviations above the mean and the usual low is approximately 2 standard deviations below.
Going in the opposite direction, on the other hand, if we know the standard deviation we can estimate:

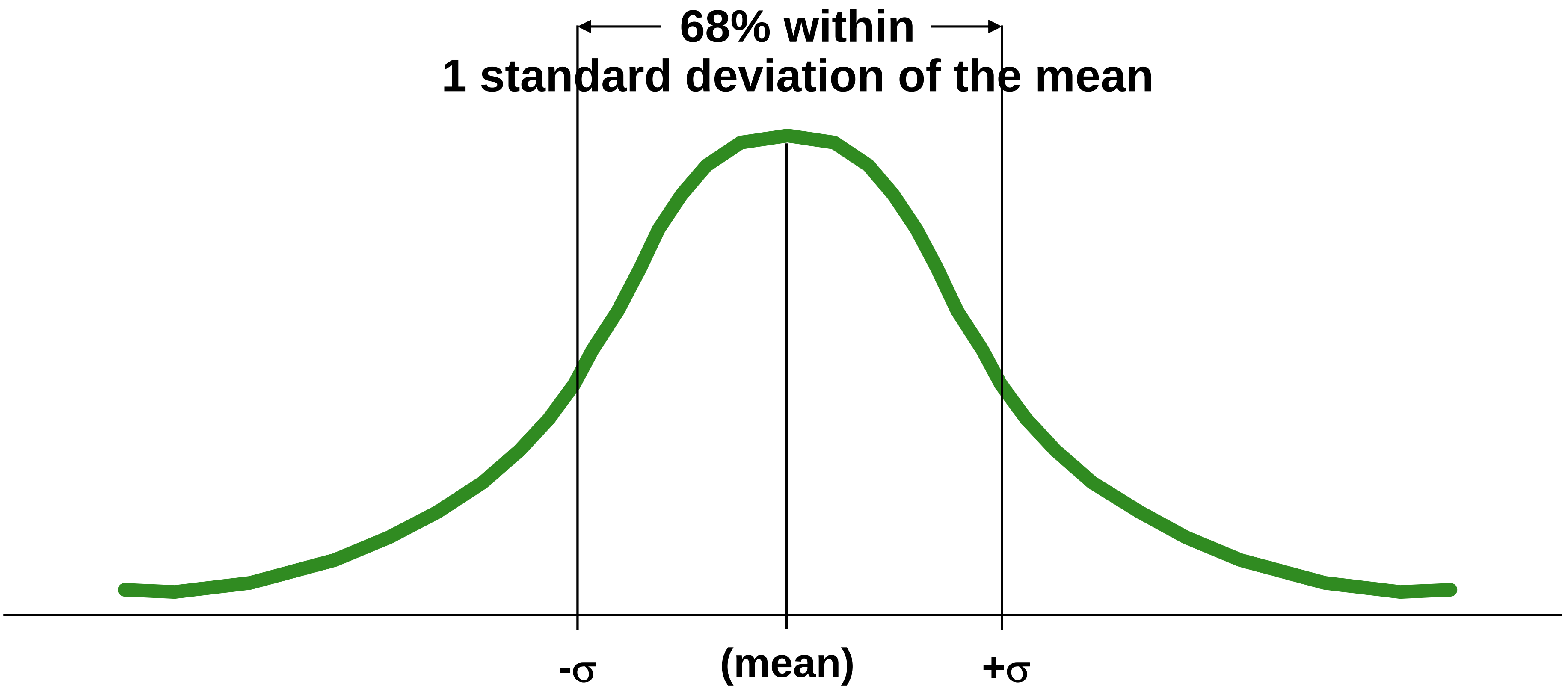
low value ≈ mean – 2 x standard deviation
high value ≈ mean + 2 x standard deviation

The 68-95-99.7 Rule for a Normal Distribution

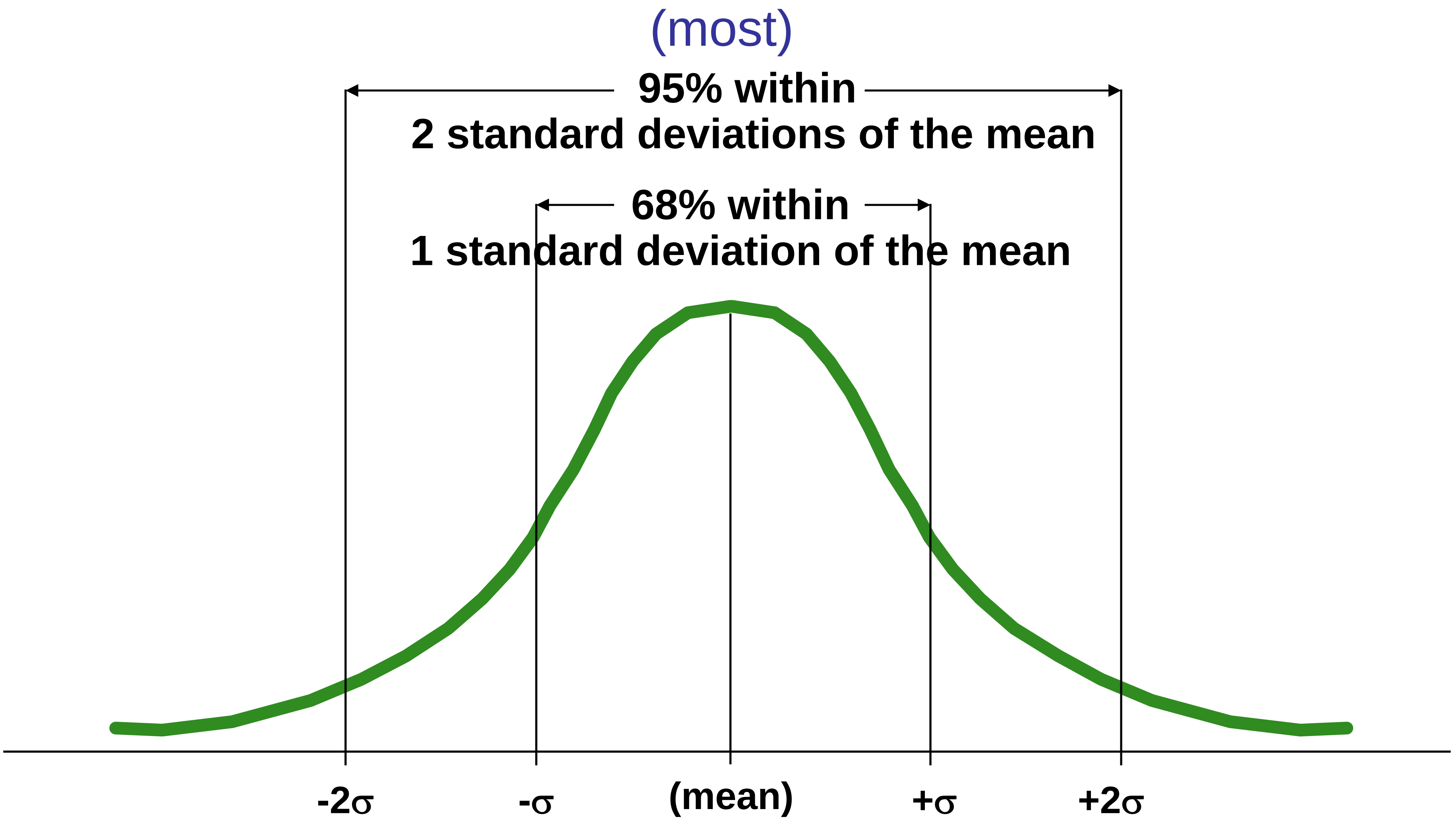


The 68-95-99.7 Rule for a Normal Distribution

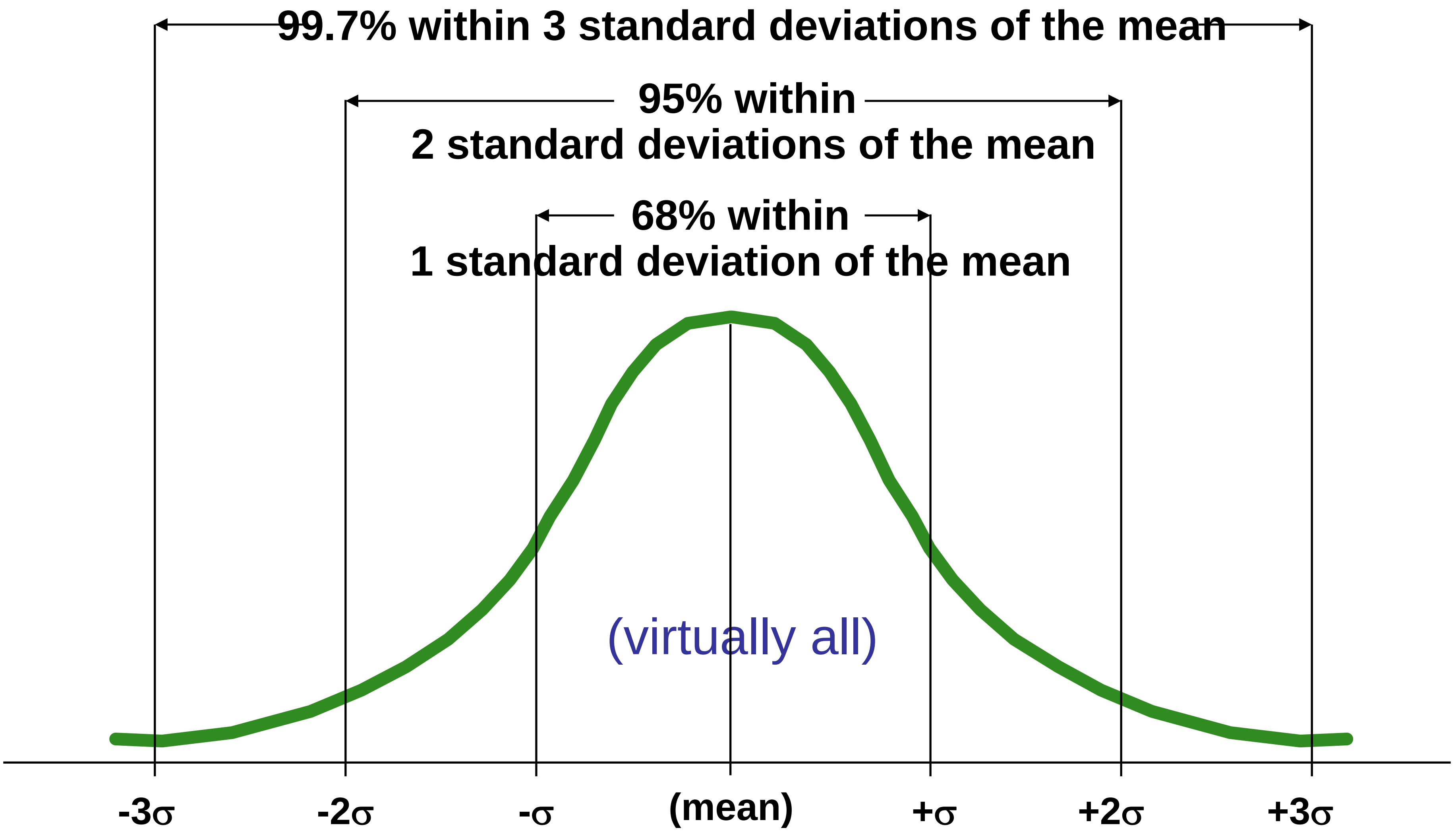
(just over two thirds)



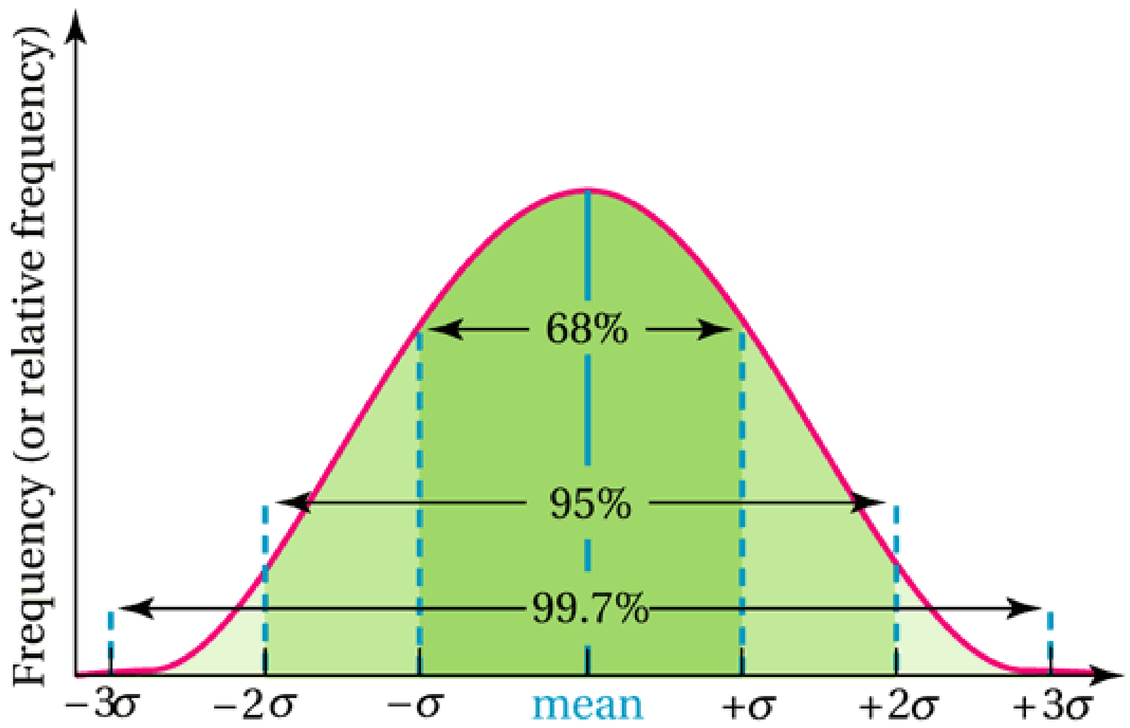
The 68-95-99.7 Rule for a Normal Distribution



The 68-95-99.7 Rule for a Normal Distribution



Also known as the Empirical Rule



Z-Score Formula

$$\text{standard score} = z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

Example:
If the nationwide ACT mean were 21 with a standard deviation of 4.7, find the *z*-score for a 30.
What does this mean?

$$z = \frac{30 - 21}{4.7} = 1.91$$

This means that an ACT score of 30 would be about 1.91 standard deviations above the mean of 21.

Standard Scores and Percentiles

TABLE 6.3 **Standard Scores and Percentiles**

Z-SCORE	PERCENTILE	Z-SCORE	PERCENTILE	Z-SCORE	PERCENTILE	Z-SCORE	PERCENTILE
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41’	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Inferential Statistics Definitions

- A set of measurements or observations in a statistical study is said to be **statistically significant** if it is unlikely to have occurred by chance.
- 0.05 (1 out of 20) or 0.01 (1 out of 100) are two very common **levels of significance** that indicate the probability that an observed difference is simply due to chance.
- At the level of 95% confidence we can estimate the **margin of error** that a sample of size *n* is different from the actual population measurement (parameter) by calculating:

$$\text{margin of error} \approx \frac{1}{\sqrt{n}}$$

- The **null hypothesis** claims a specific value for a population parameter null hypothesis: population parameter = claimed value
- The **alternative hypothesis** is the claim that is accepted if the null hypothesis is rejected.

Outcomes of a Hypothesis Test

- Rejecting the null hypothesis, in which case we have evidence that supports the alternative hypothesis.

- Not rejecting the null hypothesis, in which case we lack sufficient evidence to support the alternative hypothesis.