Using and Understanding

Mathematics

A Quantitative Reasoning Approach

Third Edition

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Chapter 7

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Probability Fundamentals

$$P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of outcomes}}$$

Example: What is the probability of getting 2 heads with 2 coin tosses?

$P(2 \text{ heads with 2 tosses}) = 1/4$ {HH}/{HH, TT, HT, TH}

$$P(\text{not } A) = 1 - P(A)$$

Example: If there is a 40% chance of rain for today, what is the chance of not having any rain?

$P(\text{no rain}) = 1 - 40\% = 60\%$

$$\text{odds for event } A = \frac{P(A)}{P(\text{not } A)}$$

Example: What are the odds for flipping 2 heads?

odds for 2 heads = $P(2 \text{ heads}) \div P(\text{not 2 heads})$
$= (1/4) \div (3/4) = 1/3$ or (1 to 3).

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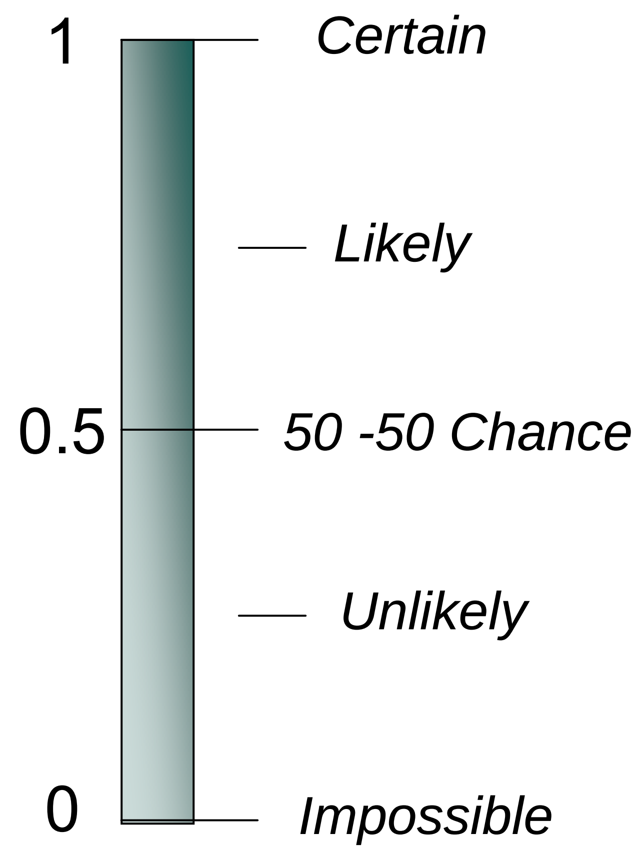
Expressing Probability

The probability of an event, expressed as *P*(event), is alwaysbetween 0 and 1 (inclusive).

A probability of 0 means the event is impossible and a probability of 1 means the event is certain.

$$0 \leq P(A) \leq 1$$



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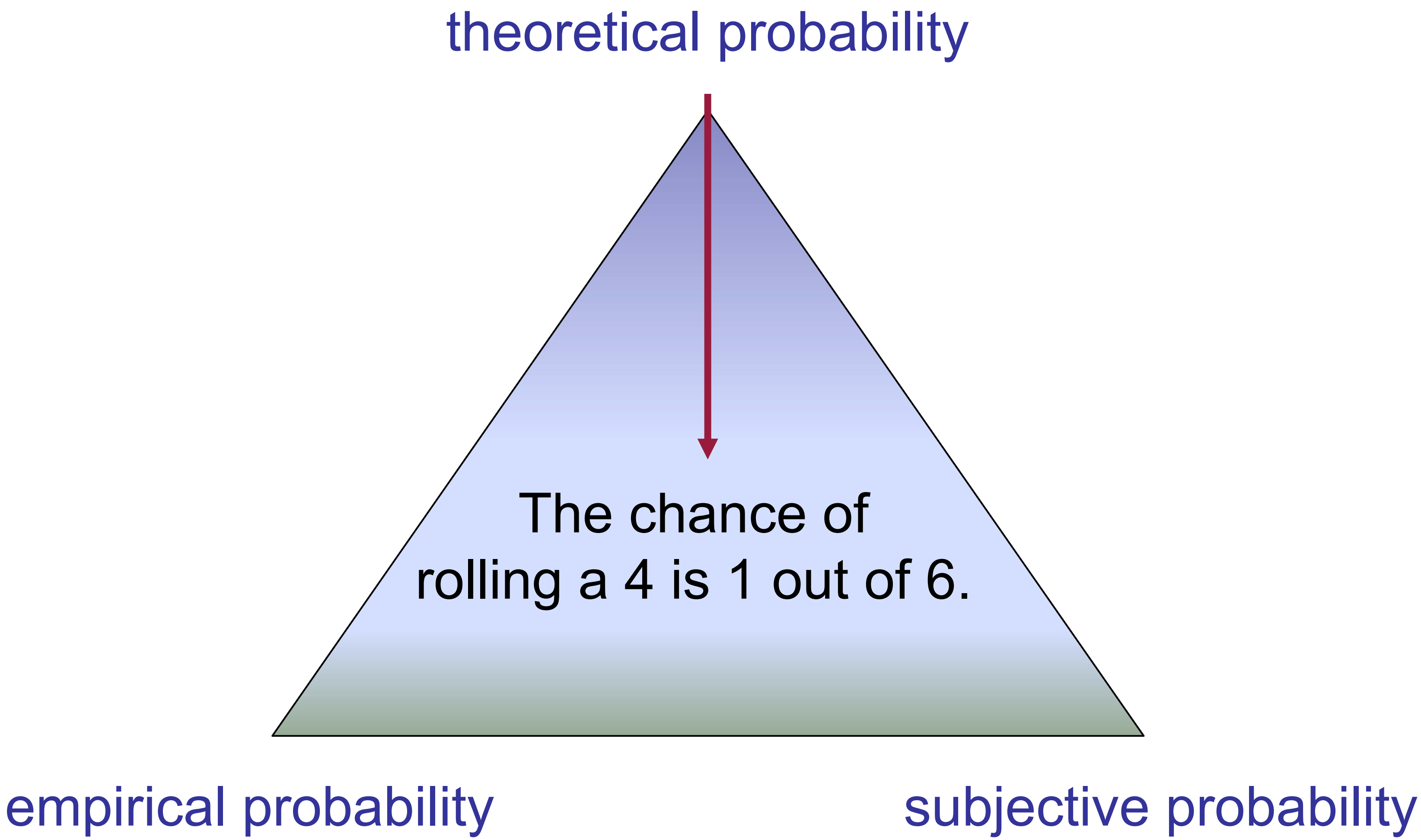
Three Types of Probabilities



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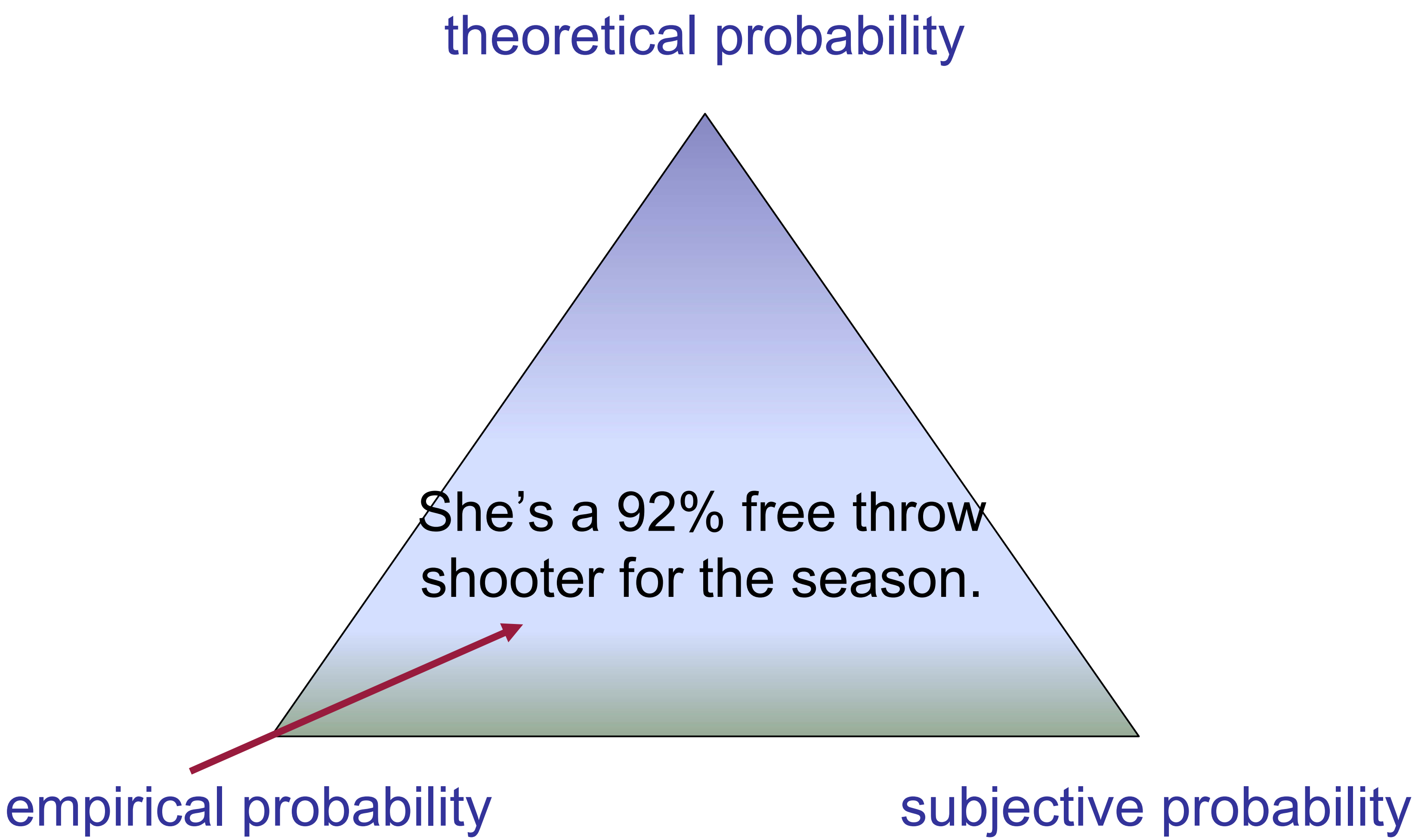

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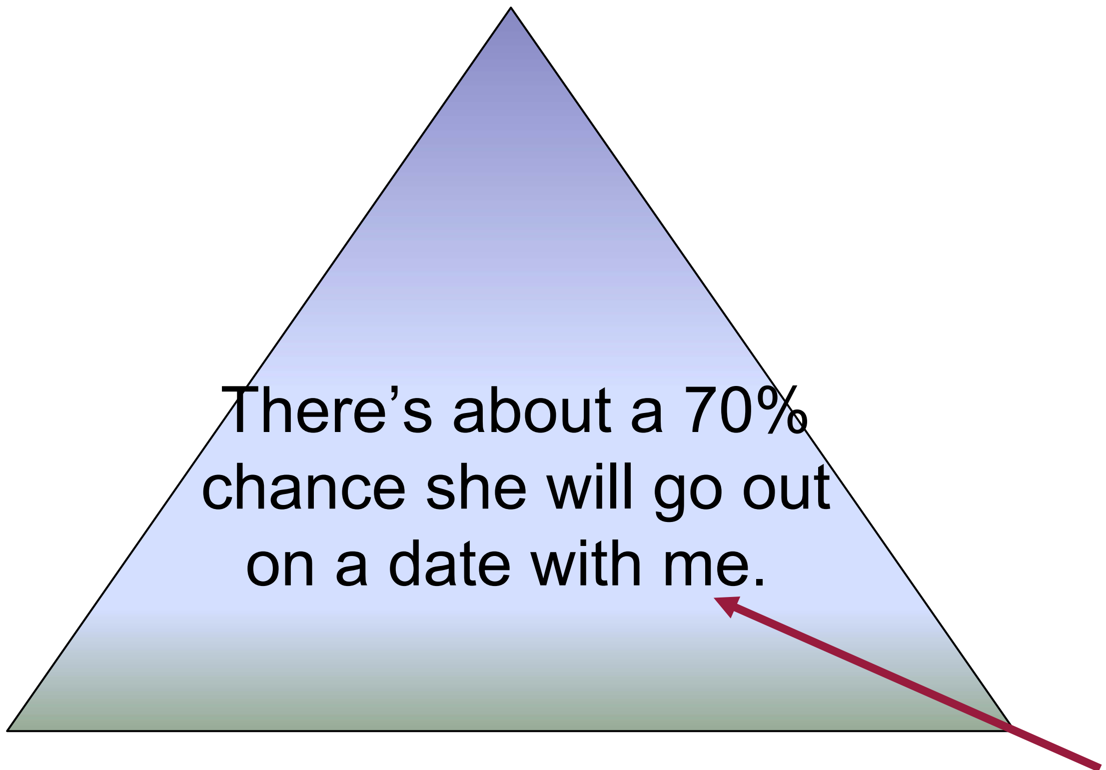
Three Types of Probabilities

theoretical probability



empirical probability

subjective probability

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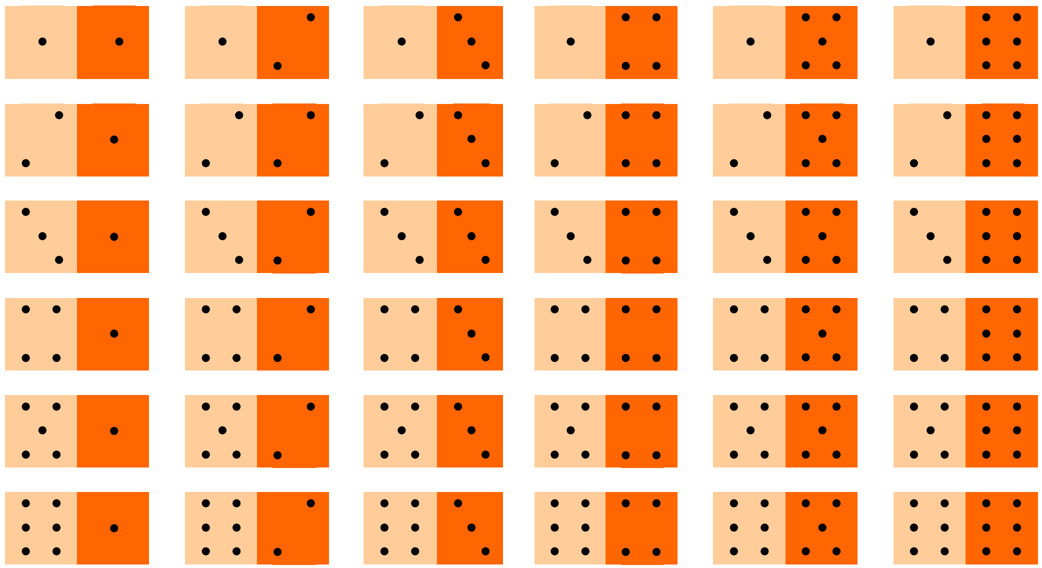
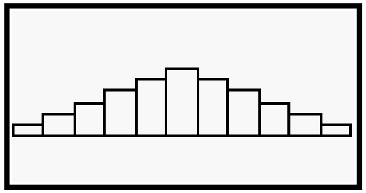
Outcomes Using Two Fair Dice



Probabilities of rolling numbers 2 through 12



	2	3	4	5	6	7	8	9	10	11	12
Fraction	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
%	2.8%	5.6%	8.3%	11.1%	13.9%	16.7%	13.9%	11.1%	8.3%	5.6%	2.8%

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Outcomes and Events

Question: Assuming equal chance of having a boy or girl at birth, what is the probability of having two girls and two boys in a family of four children?

Answer: **37.5%**

Why?: There are **16** possible outcomes with **6** ways (events) to have two of each gender. 6/16 = **37.5%**

Scenarios	Possible Combinations
All 4 Girls	{GGGG}
3 Girls and 1 Boy	{GGGB}, {GGBG}, {GBGG}, {BGGG}
2 Girls and 2 Boys	{GGBB}, {GBGB}, {GBBG} {BGBG}, {BBGG}, {BGGB}
1 Girl and 3 Boys	{GBBB}, {BGBB}, {BBGB}, {BBBG}
All 4 Boys	{BBBB}

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Combining Probabilities

Independent Events: $P(A \text{ and } B) = P(A) \bullet P(B)$

Example: P(rolling a 2 and drawing an Ace of Diamonds)

$= 1/6 \bullet 1/52 = 1/312$

Dependent Events: $P(A \text{ and } B) = P(A) \bullet P(B \text{ given } A)$

Example: P(drawing a Queen and then drawing a 5)
(assume we are not replacing the 1st card drawn)

$= 4/52 \bullet 4/51 = 16/2652 = 4/663$

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More on Combining Probabilities

Non-overlapping Events: $P(A \text{ or } B) = P(A) + P(B)$

Example: Suppose you roll a single die.
What is the probability of rolling either a 2 or a 3?

$$P(2 \text{ or } 3) = P(2) + P(3) = 1/6 + 1/6 = 2/6 = 1/3$$

Overlapping Events: $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$

Example: What is the probability that in a standard shuffled deck of cards you will draw a queen or a club?

$$= P(\text{queen or club})$$
$$= P(\text{queen}) + P(\text{club}) – P(\text{queen and club})$$
$$= 4/52 + 13/52 – 1/52 = 16/52 = 4/13$$

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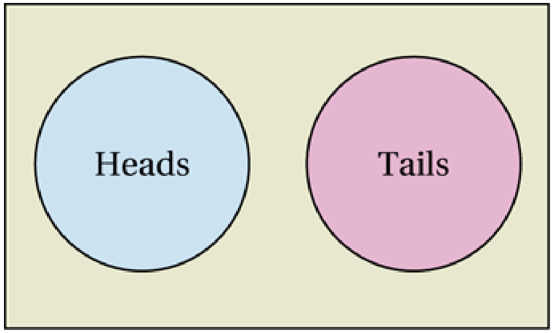
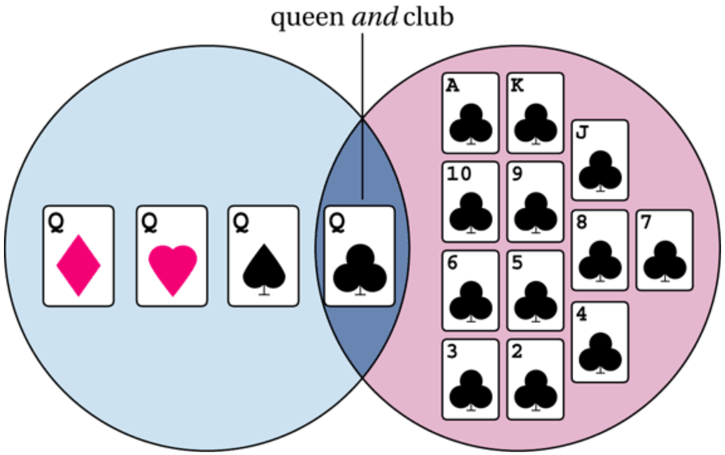
More on Combining Probabilities

Non-overlapping Events: $P(A \text{ or } B) = P(A) + P(B)$



Overlapping Events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$



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At Least Once Rule

P(at least one event A in n trials) = 1 – P(not event A in n trials)

= 1 – $P(\text{not } A)^n$

Example: What is the probability of having at least one girl in a family of four children?

$$1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$$

Scenarios	**Possible Combinations**
All 4 Girls	{GGGG}
3 Girls and 1 Boy	{GGGB}, {GGBG}, {GBGG}, {BGGG}
2 Girls and 2 Boys	{GGBB}, {GBGB}, {GBBG} {BGBG}, {BBGG}, {BGGB}
1 Girl and 3 Boys	{GBBB}, {BGBB}, {BBGB}, {BBBG}
All 4 Boys	{BBBB}

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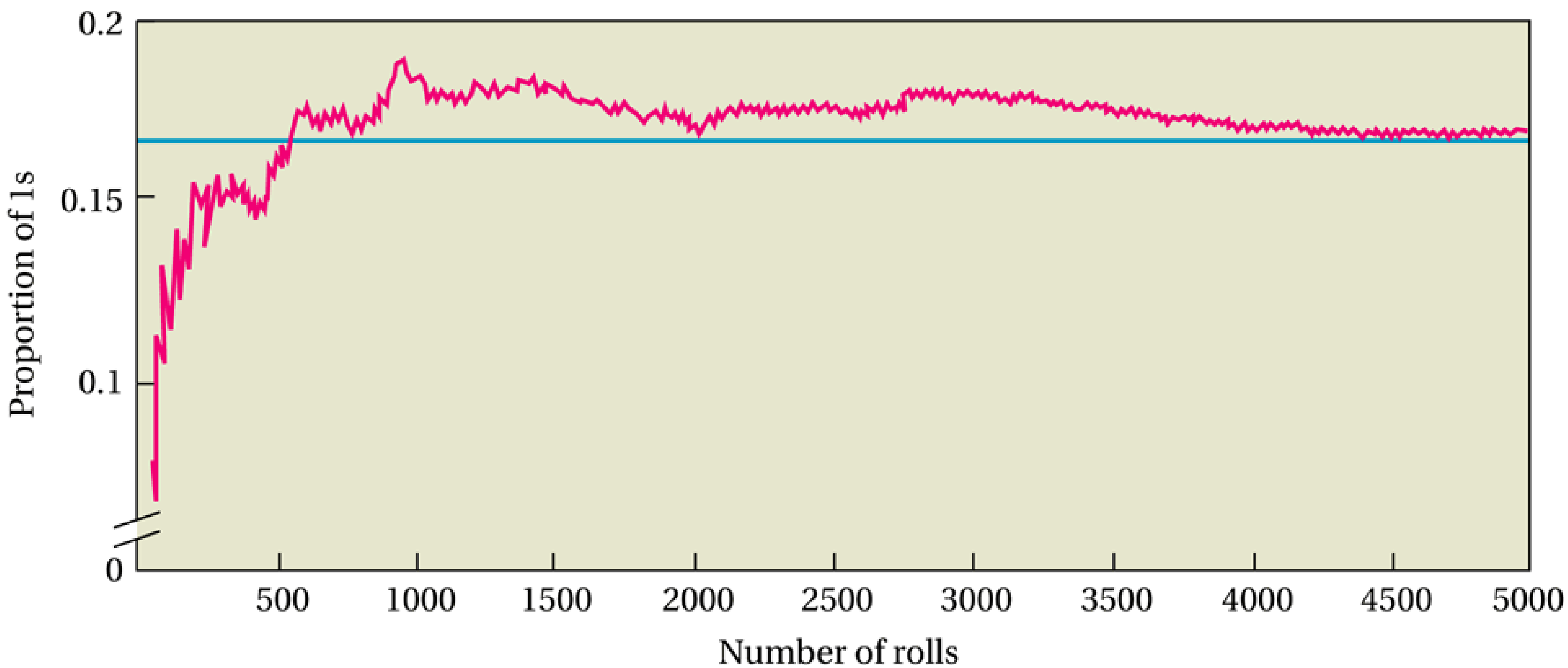
The Law of Averages

If the process is repeated through many trials, the proportion of the trials in which event *A* occurs will be close to the probability *P*(*A*). The larger the number of trials the closer the proportion should be to *P*(*A*).

(Sometimes this is referred to as the Law of Large Numbers.)



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Expected Value

Consider two events, each with its own value and probability.

Expected Value = (event 1 value) • (event 1 probability)
+ (event 2 value) • (event 2 probability)

Example: Suppose an automobile insurance company sells an insurance policy with an annual premium of $200. Based on data from past claims, the company has calculated the following probabilities:

An average of 1 in 50 policyholders will file a claim of $2,000
An average of 1 in 20 policyholders will file a claim of $1,000
An average of 1 in 10 policyholders will file a claim of $500

Assuming that the policyholder could file any of the claims above, what is the expected value to the company for each policy sold?

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Expected Value

Consider two events, each with its own value and probability.

Expected Value = (event 1 value) • (event 1 probability)
+ (event 2 value) • (event 2 probability)

Let the $200 premium be positive (income) with a probability of 1 since there will be no policy without receipt of the premium. The insurance claims will be negatives (expenses).

The expected value is

$$= \$200(1) + -\$2000(1/50) + -\$1000(1/20) + -\$500(1/10)$$
$$= \$60$$

(Suggesting that if the company sold many, many policies, on average, the return per policy is a positive $60.)

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The Gambler's Fallacy

The gambler's fallacy is the mistaken belief that a streak of bad luck makes a person "due" for a streak of good luck.

Example (*continued losses*):

Suppose you are playing the coin toss game, in which you win \$1 for heads and lose \$1 for tails. After 100 tosses you are \$10 in the hole because you flip perhaps 45 heads and 55 tails. The empirical probability is .45 for heads.

. . . So you keep playing the game:

With 1,000 tosses perhaps you get 490 heads and 510 tails. The empirical ratio is now .49 but you are \$20 in the hole at this point. Because you know the Law of Averages helps us to know that the eventual theoretical probability is .50, you decide to . . .

. . . play the game just a *little* more:

After 10,000 tosses perhaps you flip 4,985 heads and 5,015 tails. The empirical ratio is now .4985 but you are \$30 in the hole even though the ratio is approaching the hypothetical 50%.

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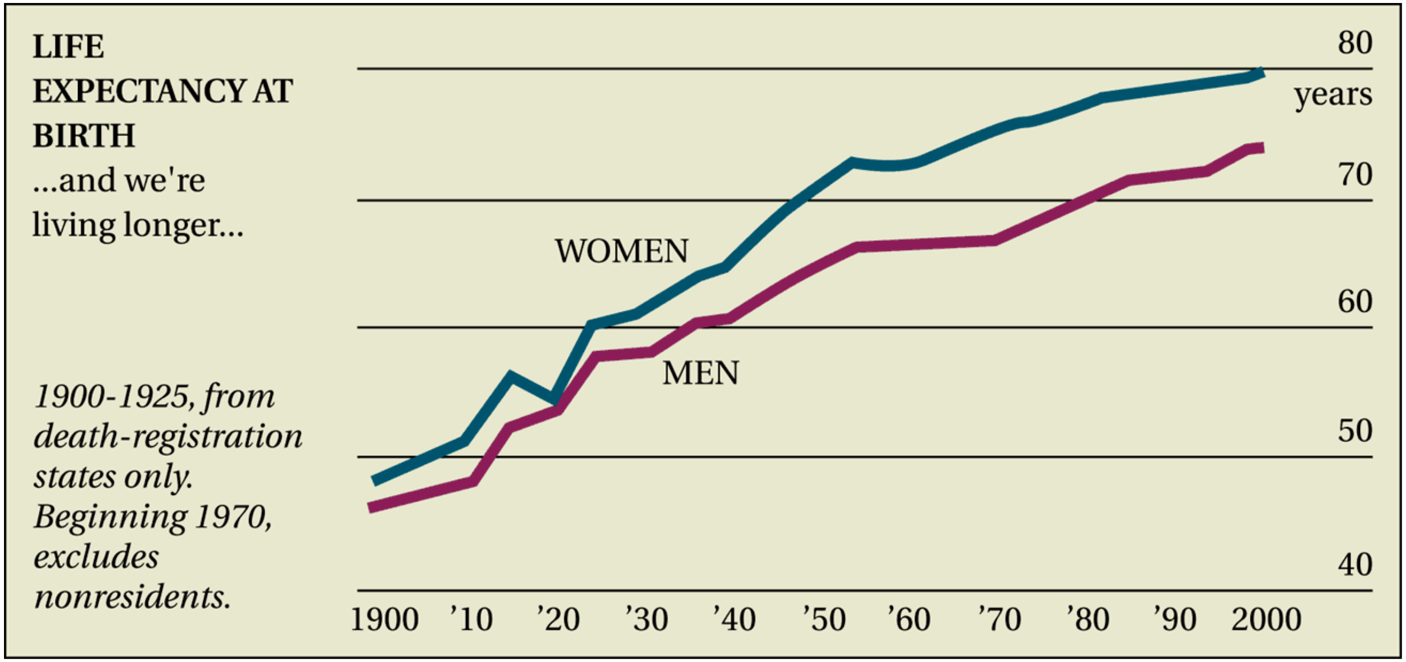
Life Expectancy at Birth



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Arrangements with Repetition

If we make r selections from a group of n choices, a total of n^r different arrangements are possible.

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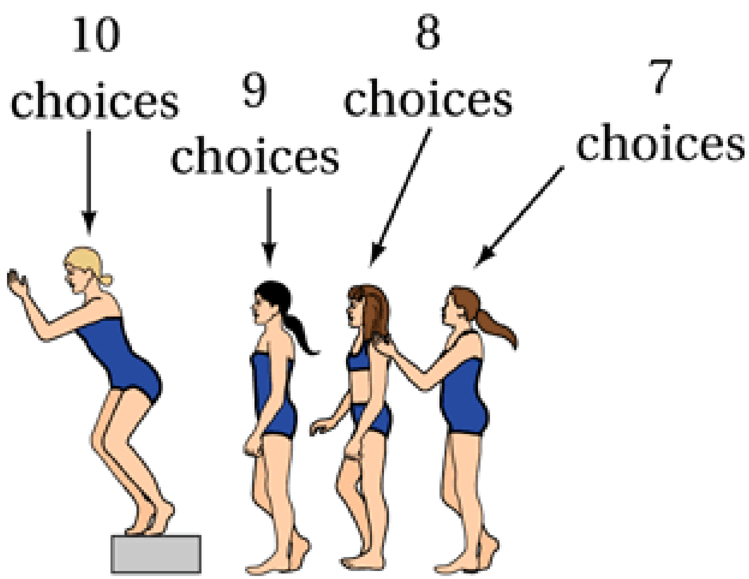
The Permutations Formula

If we make *r* selections from a group of *n* choices, the number of permutations (arrangements in which order matters) is

$$_nP_r = \frac{n!}{(n-r)!}$$



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The Permutations Formula

If we make r selections from a group of n choices, the number of permutations (arrangements in which order matters) is

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Question:

If an international track event has 8 athletes participating and three medals (gold, silver and bronze) are to be awarded, how many different orderings of the top three athletes are possible?

Answer: **336** **Why?**

$$n = 8;\ r = 3; \qquad {}_8P_3 = \frac{8!}{(8-3)!} = 336$$

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The Combinations Formula

If we make *r* selections from a group of *n* items, the number of combinations (arrangements in which order does not matter) is

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} $$

Question:

If a committee of 3 people are needed out of 8 possible candidates and there is not any distinction between committee members, how many possible committees would there be?

Answer: **56** **Why?**

$$ n = 8;\ r = 3; \qquad {}_8C_3 = \frac{8!}{(8-3)! \cdot 3!} = 56 $$

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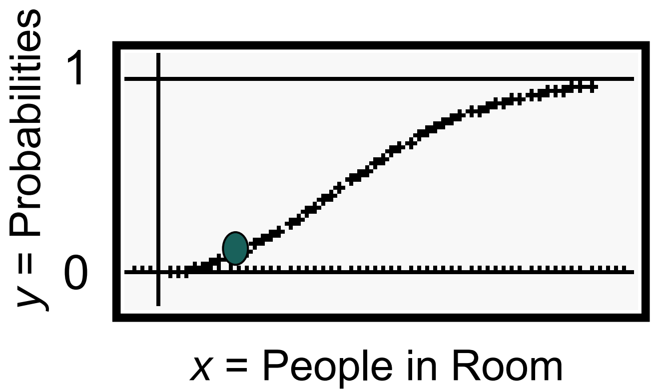
Birthday Coincidence

What are the probabilities that someone in a room of *x* people will have a birthday in common with someone else in that room?



x = People in Room:	10	15	20	25	30	35	40	45
y = Probabilities:	.117	.253	.411	.569	.706	.814	.891	.940

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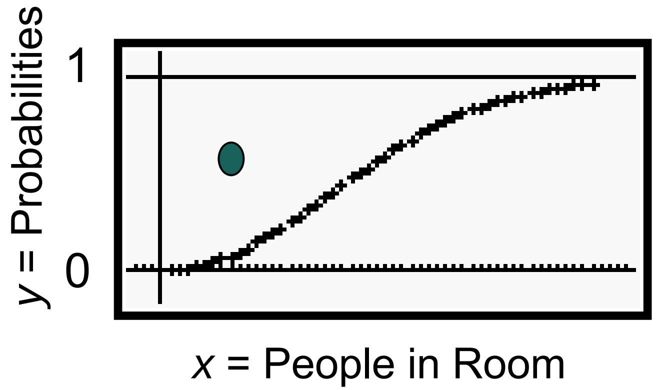

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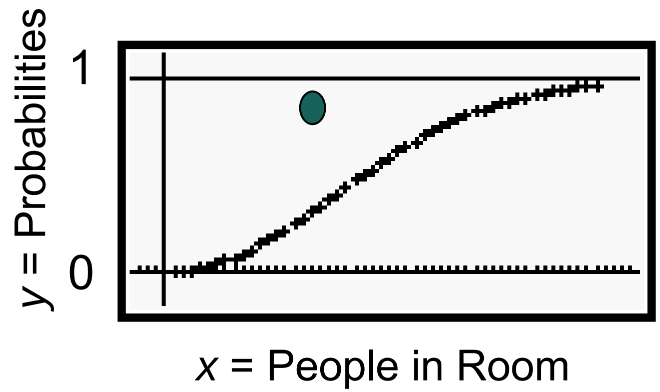

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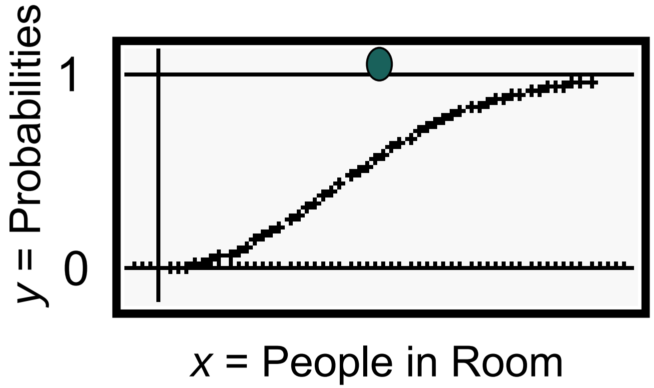

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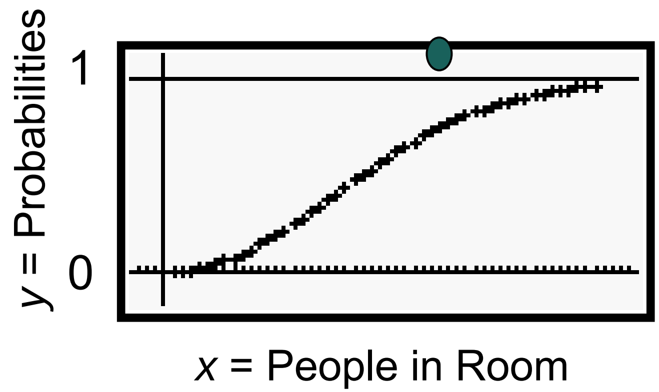

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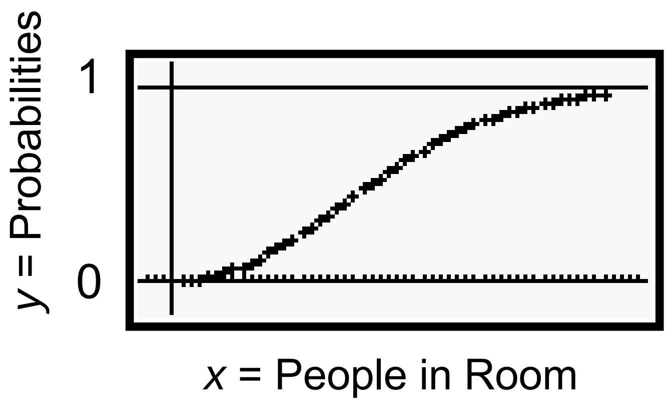

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How many people in the room would be required for 100% certainty?

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