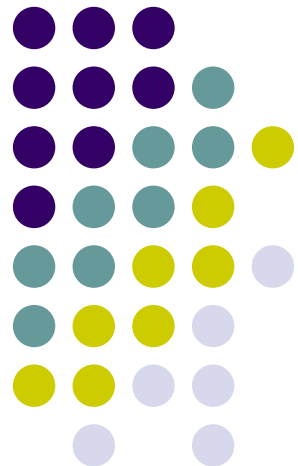
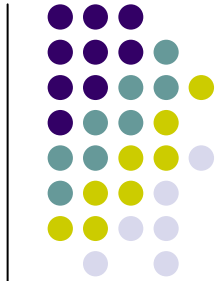
Discrete Probability Distributions

MATH 1342 Elementary Statistics



Why Study Probability?

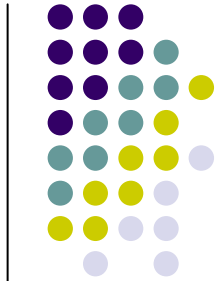


- Probability helps us make informed decisions in a variety of areas.
  - Finance: does an investment have a chance of making a profit?
  - Law: does a pool of jurors represent a fair cross-section of Americans?
  - Medicine: does a vaccine help more people than hurt?

Why Be Discrete?

- A *discrete* quantity is usually represented by a whole number (0,1,2, etc.). Discrete data usually does not use fractions or decimals.
- In the last slide, the variables in question are a discrete quantities.
  - How much should invest? $100, $200, or more?
  - How many people of one demographic were on the jury panel? 3,4, or more?
  - How many people were injured by a vaccine? 5,6, or more?

Probability Distributions



- A discrete probability distribution table lists
  - all possible values for a discrete random variable X ; and
  - corresponding probabilities $P(x)$ for each value

Application: Return on Investment

- How much return on investment (profit or loss) can we expect from:
  - Life Insurance
  - Stocks, Bonds
  - Lotteries & other games of chance

Important Computations

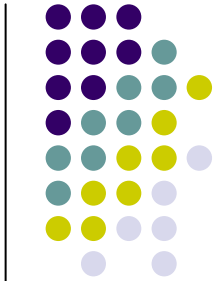
- Know how to verify if a table of probabilities is a discrete probability distribution
  - **$\Sigma P(x) = 1$** and **$0 \leq P(x) \leq 1$**
- Know how to compute the mean **μ** and standard deviation **σ** of discrete probability distribution
  - Use the TI calculator:

    STAT **[Calc] 1-Var Stats L1,L2**

Applied Solution Strategies

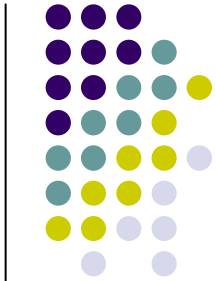
- Organize your possible earnings and corresponding probabilities into a probability distribution table
- Compute the mean value (aka *expected* value) to see how much you can expect to earn or lose on a given investment
- On games of chance, the expected value is always negative (i.e., we should expect to lose $$) ☹

Binomial Probability



- A very common type of discrete probability used in a variety of applications throughout the remainder of this course
- Does the jury panel have too many or too few members of a certain demographic?

Binomial Vocabulary



- A binomial experiment or event is called a *trial*. Flipping a coin is the classic binomial trial.
- Each trial is *independent* of other trials. One flip of the coin does not affect another.
- Each trial has 2 *mutually exclusive* outcomes. The coin will land heads or tails, but not both!
- The probability of each outcome remains constant.

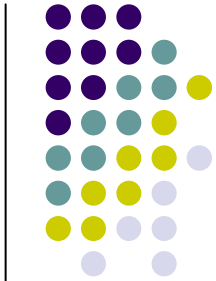
More Binomial Vocabulary

- Binomial probability distributions have a finite number of trials, called ***n***. If we flip a coin 20 times, then ***n* = 20**
- The number of times a trial has a desired outcome (e.g. tails) is represented by ***x***. Note that **x ≤ n**
- The probability of a trial having a desired outcome (e.g. tails) is represented by ***p***
- The probability of a trial having the opposite outcome (e.g. heads) is represented by ***1-p*** or ***q***

Computing Binomial Probability

- Use the TI calculator: 2nd [DISTR] **binompdf(n,p,x)**
- The TI calculator **binomcdf** is a useful shortcut for *cumulative* probability (automatically adding up all probabilities up to a specified *x*-value)
- Microsoft Excel has similar features under the **BINOMDIST** function

Algebra is Really Cool



- Through use of college algebra, we can derive two remarkable shortcut formulas for the mean and standard deviation of a binomial distribution

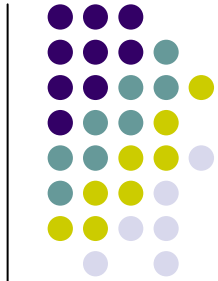
$$\mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)}$$

- The longer formulas for mean and standard deviation are not necessary with binomial problems

Binomial is the New Normal

- A very interesting pattern emerges if we express a binomial probability distribution as a histogram
- As the number of trials ***n*** grows larger, the histogram becomes follows a *normal* or bell-shaped pattern (as guaranteed by the Central Limit Theorem)
- About 95% of a binomial distribution falls between ***μ-2σ*** and ***μ+2σ***. Anything outside this range is considered an unusually small or large result.

Binomial Application



- In a certain Texas county, the population was 79.1% Hispanic. The county jury pool of 870 potential jurors was supposed to be selected at random. Only 339 potential jurors were Hispanic. Do these numbers show evidence of discrimination? (from *Castaneda v. Partida*, a 1977 Supreme Court case)

Common Mistakes to Avoid

- For binomial problems, make sure the total sample size ***n*** is not more than 5-10% of the total population. If not, one trial may not be independent of the next trial.
- For binomial problems, make sure ***x*** and ***p*** measure the same event or quantity
- Know how to translate spoken language into Math: e.g. *no more than* → $\leq$ or *at least* → $\geq$

Poisson Distribution

- Useful to measure probability over an interval of time
  - What is the probability of a certain number of people entering a queue (e.g. at grocery store, drive-thru)?
  - What is the probability that a vaccine may cause fatalities over a specified interval of time?
- Useful way to approximate the binomial probability without the aid of an advanced calculator for experiments with very large $n \geq 100$.

Computing Poisson Probability

- Use the TI calculator: 2nd [DISTR] **poissonpdf(μ,x)**
  - An optional **poissoncdf(μ,x)** is available on the TI for cumulative probability.
  - Microsoft Excel has a similar POISSON function
- Note that Poisson probability does not depend on knowing an ***n*** or a ***p*** as with binomial probability.
- You only need to compute ***μ***, the average number times the event occurs in the given interval.

Summary: Know How to …

- Compute expected value for a probability distribution table
- Compute a binomial probability
- Compute the mean and standard deviation of a binomial probability
- Recognize discrete data vs. non-discrete or *continuous* data, which will be discussed in the next chapter