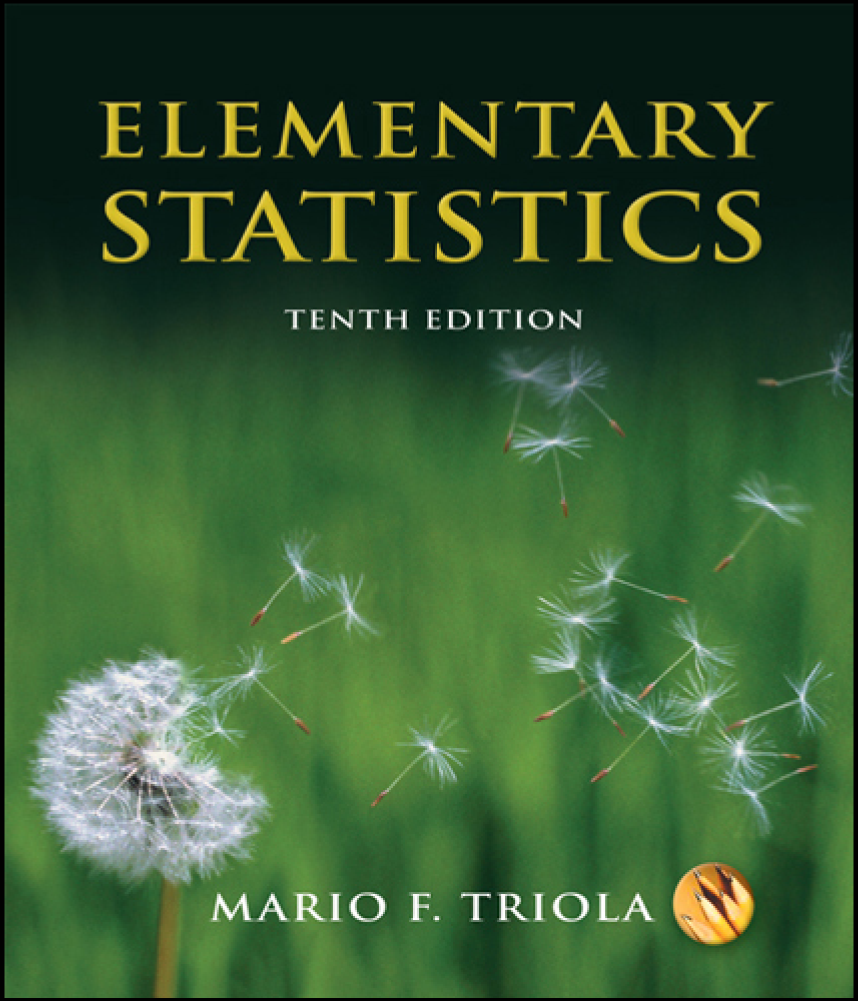
# Lecture Slides




*Elementary Statistics*
Tenth Edition

and the Triola Statistics Series

by Mario F. Triola

# Chapter 4
# Probability

# Section 4-1
# Overview

Created by Tom Wegleitner, Centreville, Virginia

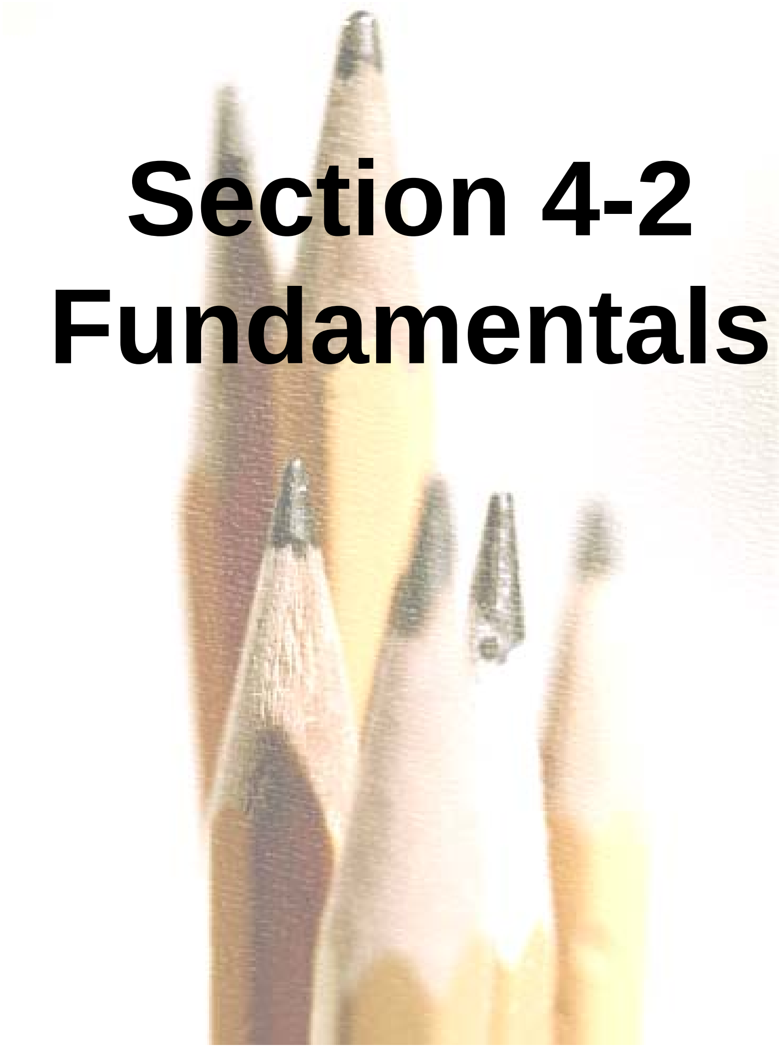
# Overview

## Rare Event Rule for Inferential Statistics:

**If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.**

**Statisticians use the rare event rule for inferential statistics.**

# Section 4-2
# Fundamentals



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# Key Concept

**This section introduces the basic concept of the probability of an event. Three different methods for finding probability values will be presented.**

**The most important objective of this section is to learn how to interpret probability values.**

# Definitions

- **Event**

  any collection of results or outcomes of a procedure

- **Simple Event**

  an outcome or an event that cannot be further broken down into simpler components

- **Sample Space**

  for a procedure consists of all possible simple events; that is, the sample space consists of all outcomes that cannot be broken down any further

# Notation for Probabilities

***P*** **- denotes a probability.**

***A*, *B*, and *C*** **- denote specific events.**

***P* (*A*)** **- denotes the probability of event *A* occurring.**

# Basic Rules for Computing Probability

**Rule 1: Relative Frequency Approximation of Probability**

**Conduct (or observe) a procedure, and count the number of times event *A* actually occurs. Based on these actual results, *P*(*A*) is estimated as follows:**

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{number of times trial was repeated}}$$

# Basic Rules for Computing Probability - cont

## Rule 2: Classical Approach to Probability

**(Requires Equally Likely Outcomes)**

**Assume that a given procedure has *n* different simple events and that each of those simple events has an equal chance of occurring. If event *A* can occur in *s* of these *n* ways, then**

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

# Basic Rules for Computing Probability - cont

## Rule 3: Subjective Probabilities

***P*(*A*), the probability of event *A*, is estimated by using knowledge of the relevant circumstances.**

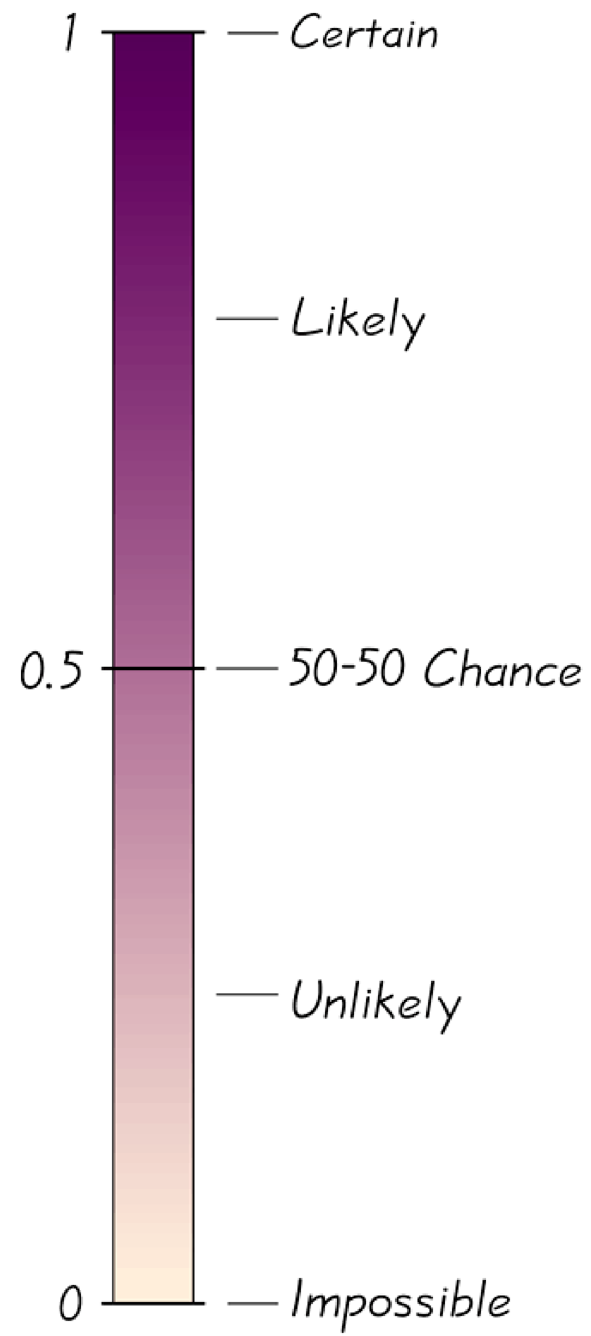
# Law of Large Numbers

**As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.**

# Probability Limits

- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- For any event *A*, the probability of *A* is between 0 and 1 inclusive.
  That is, $0 \leq P(A) \leq 1$.

# Possible Values for Probabilities



# Definition

**The complement of event *A*, denoted by $\bar{A}$, consists of all outcomes in which the event *A* does not occur.**

# Rounding Off Probabilities

When expressing the value of a probability, either give the **exact** fraction or decimal or round off final decimal results to three significant digits. (**Suggestion**: When the probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.)

# Definitions

The **actual odds against** event *A* occurring are the ratio $P(A)/P(\overline{A})$, usually expressed in the form of **a:b** (or "**a** to **b**"), where *a* and *b* are integers having no common factors.

**The actual odds in favor** of event *A* occurring are the reciprocal of the actual odds against the event. If the odds against *A* are **a:b**, then the odds in favor of *A* are **b:a**.

**The payoff odds** against event *A* represent the ratio of the net profit (if you win) to the amount bet.

**payoff odds against event**
***A* = (net profit) : (amount bet)**

# Recap

**In this section we have discussed:**

- **Rare event rule for inferential statistics.**
- **Probability rules.**
- **Law of large numbers.**
- **Complementary events.**
- **Rounding off probabilities.**
- **Odds.**

# Section 4-3
# Addition Rule

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# Key Concept

**The main objective of this section is to present the addition rule as a device for finding probabilities that can be expressed as *P*(*A* or *B*), the probability that either event *A* occurs or event *B* occurs (or they both occur) as the single outcome of the procedure.**

# Definition

## Compound Event

**any event combining 2 or more simple events**

## Notation

***P*(*A* or *B*) = *P* (in a single trial, event *A* occurs or event *B* occurs or they both occur)**

# General Rule for a Compound Event

**When finding the probability that event *A* occurs or event *B* occurs, find the total number of ways *A* can occur and the number of ways *B* can occur, but find the total in such a way that no outcome is counted more than once.**

# Compound Event

## Formal Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

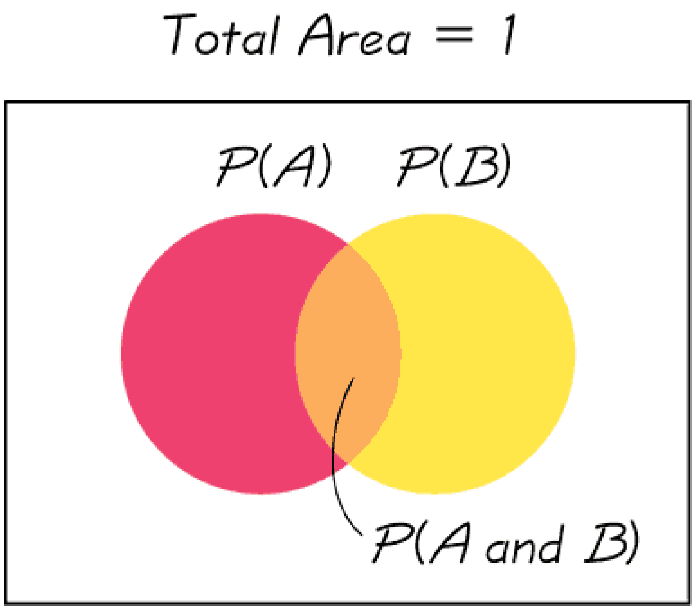
**where $P(A \text{ and } B)$ denotes the probability that $A$ and $B$ both occur at the same time as an outcome in a trial or procedure.**

## Intuitive Addition Rule

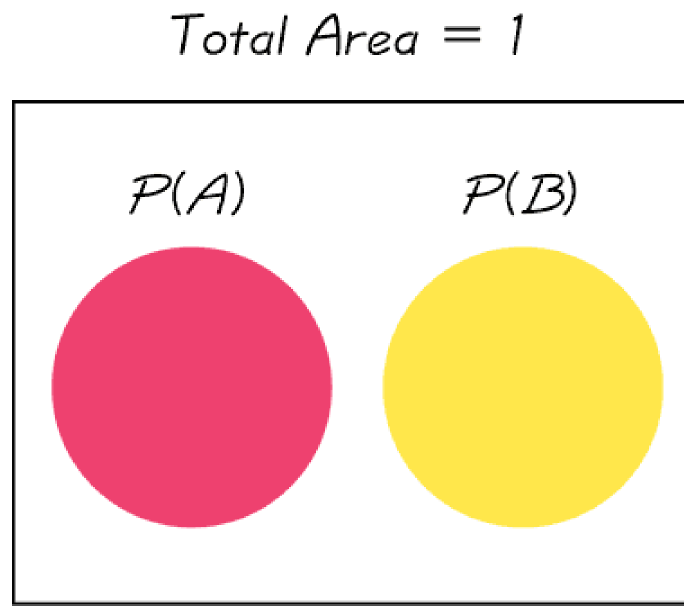
**To find $P(A \text{ or } B)$, find the sum of the number of ways event $A$ can occur and the number of ways event $B$ can occur, adding in such a way that every outcome is counted only once. $P(A \text{ or } B)$ is equal to that sum, divided by the total number of outcomes in the sample space.**

# Definition

**Events *A* and *B* are disjoint (or mutually exclusive) if they cannot occur at the same time. (That is, disjoint events do not overlap.)**




**Venn Diagram for Events That Are Not Disjoint**




**Venn Diagram for Disjoint Events**

# Complementary Events

$P(A)$ and $P(\bar{A})$ are disjoint

**It is impossible for an event and its complement to occur at the same time.**

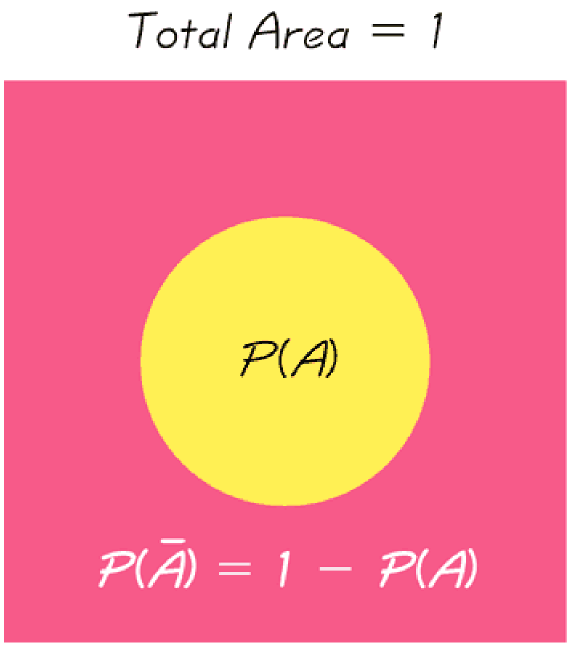
# Rules of Complementary Events

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

# Venn Diagram for the Complement of Event *A*



# Recap

**In this section we have discussed:**

- **Compound events.**
- **Formal addition rule.**
- **Intuitive addition rule.**
- **Disjoint events.**
- **Complementary events.**

# Section 4-4
# Multiplication Rule: Basics

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# Key Concept

If the outcome of the first event *A* somehow affects the probability of the second event *B*, it is important to adjust the probability of *B* to reflect the occurrence of event *A*.

The rule for finding *P*(*A* and *B*) is called the multiplication rule.

# Notation

***P*(*A* and *B*) =**

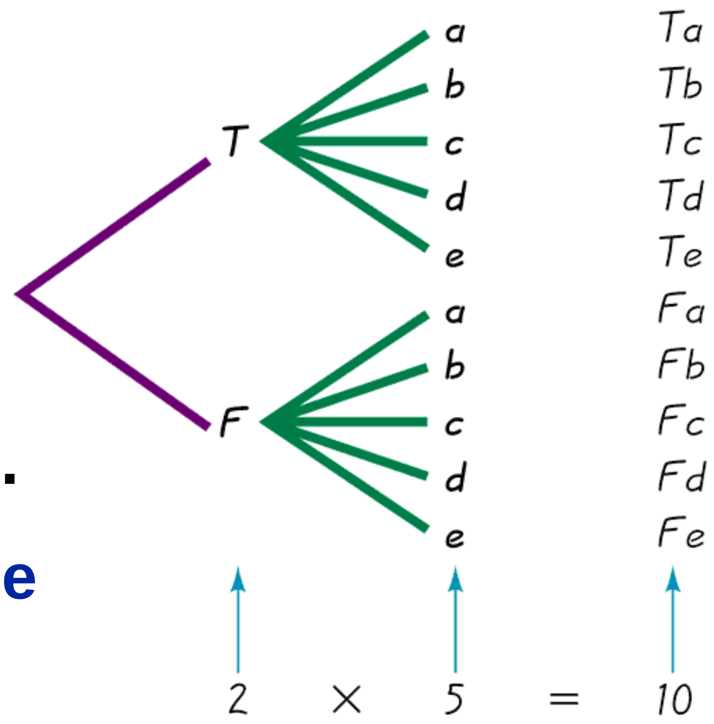
***P*(event *A* occurs in a first trial and event *B* occurs in a second trial)**

# Tree Diagrams

A **tree diagram** is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are helpful if the number of possibilities is not too large.

This figure summarizes the possible outcomes for a true/false followed by a multiple choice question.

Note that there are 10 possible combinations.



# Key Point – Conditional Probability

**The probability for the second event *B* should take into account the fact that the first event *A* has already occurred.**

# Notation for Conditional Probability

$P(B|A)$ represents the probability of event $B$ occurring after it is assumed that event $A$ has already occurred (read $B|A$ as "$B$ given $A$.")

# Definitions

## Independent Events

**Two events *A* and *B* are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of occurrence of the others.) If *A* and *B* are not independent, they are said to be dependent.**

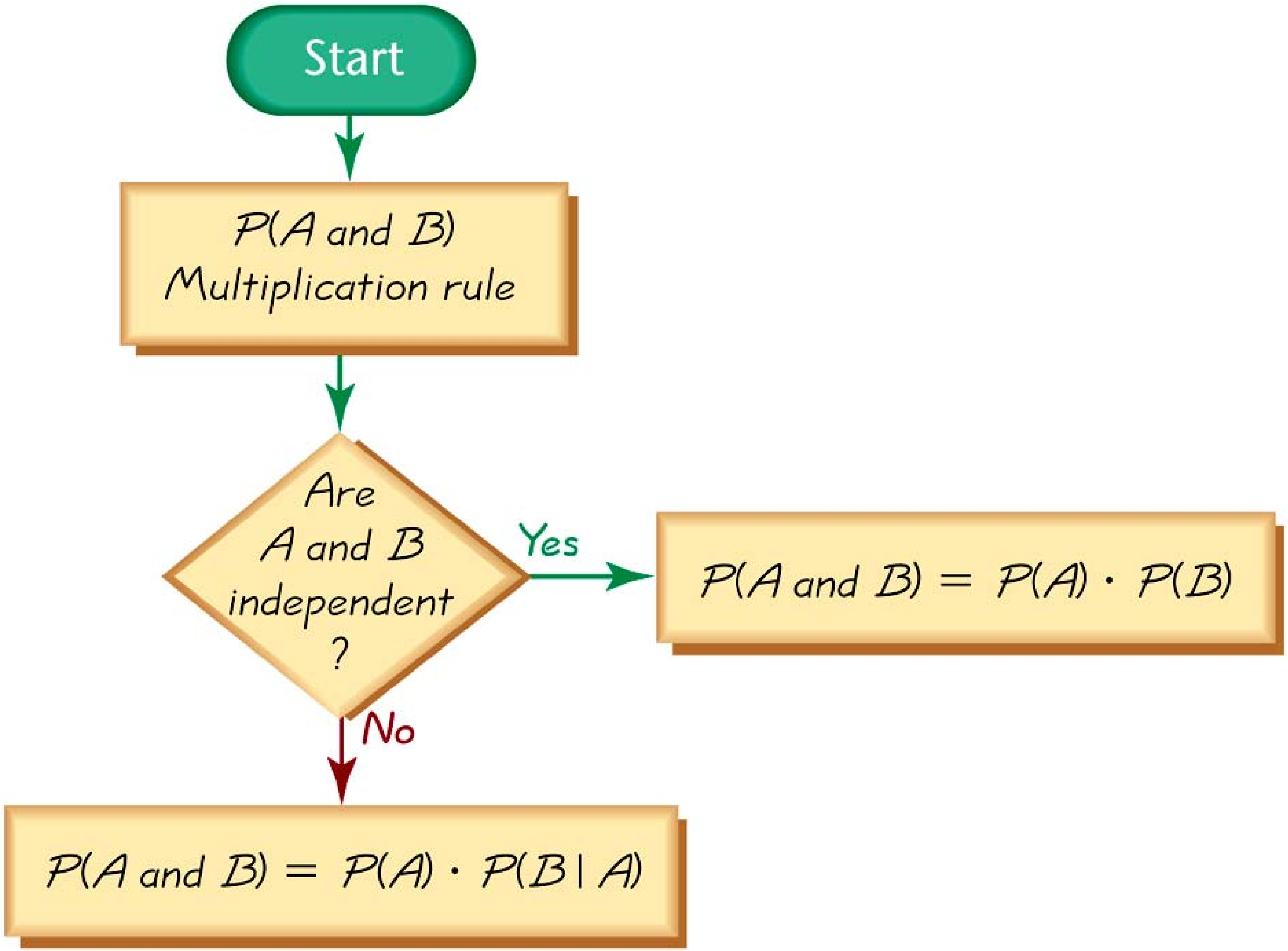
# Formal Multiplication Rule

- $P(A \text{ and } B) = P(A) \cdot P(B|A)$
- Note that if A and B are independent events, P(B|A) is really the same as P(B).

# Intuitive Multiplication Rule

**When finding the probability that event *A* occurs in one trial and event *B* occurs in the next trial, multiply the probability of event *A* by the probability of event *B*, but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.**

# Applying the Multiplication Rule



# Small Samples from Large Populations

**If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).**

# Summary of Fundamentals

- In the addition rule, the word "or" in *P*(*A* or *B*) suggests addition. Add *P*(*A*) and *P*(*B*), being careful to add in such a way that every outcome is counted only once.

- In the multiplication rule, the word "and" in *P*(*A* and *B*) suggests multiplication. Multiply *P*(*A*) and *P*(*B*), but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

# Recap

**In this section we have discussed:**

- **Notation for *P*(*A* and *B*).**
- **Tree diagrams.**
- **Notation for conditional probability.**
- **Independent events.**
- **Formal and intuitive multiplication rules.**

# Section 4-5
# Multiplication Rule: Complements and Conditional Probability

Created by Tom Wegleitner, Centreville, Virginia

# Key Concept

**In this section we look at the probability of getting at least one of some specified event; and the concept of conditional probability which is the probability of an event given the additional information that some other event has already occurred.**

# Complements: The Probability of “At Least One”

- “At least one” is equivalent to “one or more.”
- The complement of getting at least one item of a particular type is that you get **no** items of that type.

# Key Principle

**To find the probability of at least one of something, calculate the probability of none, then subtract that result from 1. That is,**

$$P(\text{at least one}) = 1 - P(\text{none}).$$

# Definition

**A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event $B$ occurring, given that event $A$ has already occurred, and it can be found by dividing the probability of events $A$ and $B$ both occurring by the probability of event $A$:**

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

# Intuitive Approach to Conditional Probability

**The conditional probability of *B* given *A* can be found by assuming that event *A* has occurred and, working under that assumption, calculating the probability that event *B* will occur.**

# Recap

In this section we have discussed:

- Concept of "at least one."
- Conditional probability.
- Intuitive approach to conditional probability.

# Section 4-6
# Probabilities Through Simulations

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Addison Wesley

# Key Concept

In this section we introduce a very different approach for finding probabilities that can overcome much of the difficulty encountered with the formal methods discussed in the preceding sections of this chapter.

# Definition

A **simulation** of a procedure is a process that behaves the same way as the procedure, so that similar results are produced.

# Simulation Example

**Gender Selection** When testing techniques of gender selection, medical researchers need to know probability values of different outcomes, such as the probability of getting at least 60 girls among 100 children. Assuming that male and female births are equally likely, describe a simulation that results in genders of 100 newborn babies.

# Simulation Examples

## Solution 1:

- Flipping a fair coin 100 times where heads = female
  tails = male

| H | H | T | H | T | T | H | H | H | H |
|---|---|---|---|---|---|---|---|---|---|
| female | female | male | female | male | male | male | female | female | female |

## Solution 2:

- Generating 0’s and 1’s with a computer or calculator where
  0 = male
  1 = female

| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| male | male | female | male | female | female | female | male | male | male |

# Random Numbers

In many experiments, random numbers are used in the simulation of naturally occurring events. Below are some ways to generate random numbers.

- A table of random of digits
- STATDISK
- Minitab
- Excel
- TI-83 Plus calculator

# Random Numbers - cont

**STATDISK**

| Row | 1 Ran... |
|---|---|
| 1 | 7 |
| 2 | 8 |
| 3 | 16 |
| 4 | 38 |
| 5 | 42 |
| 6 | 46 |
| 7 | 68 |
| 8 | 68 |
| 9 | 104 |
| 10 | 117 |
| 11 | 140 |
| 12 | 195 |
| 13 | 204 |
| 14 | 244 |
| 15 | 271 |
| 16 | 274 |

**Minitab**

| ↓ | C1 | C2 |
|---|---|---|
| | | |
| 1 | 38 | |
| 2 | 48 | |
| 3 | 59 | |
| 4 | 71 | |
| 5 | 101 | |
| 6 | 107 | |
| 7 | 122 | |
| 8 | 129 | |
| 9 | 153 | |
| 10 | 153 | |
| 11 | 163 | |

# Random Numbers - cont

**Excel**

| | A |
|---|---|
| 1 | 15 |
| 2 | 3 |
| 3 | 15 |
| 4 | 362 |
| 5 | 164 |
| 6 | 184 |
| 7 | 158 |
| 8 | 59 |
| 9 | 143 |
| 10 | 85 |
| 11 | 134 |

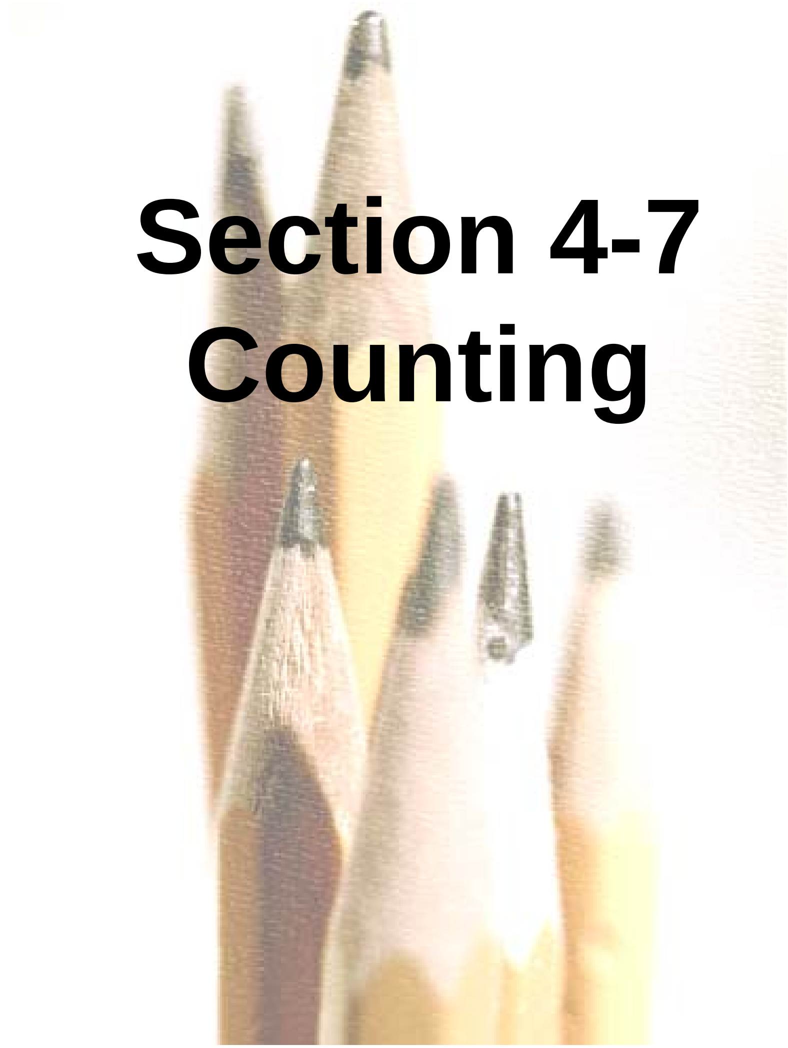
**TI-83 Plus calculator**

```
randInt(1,365,25
→L1
{79 206 340 133…
SortA(L1)
                Done
L1
{17 34 46 70 79…
```

# Recap

**In this section we have discussed:**

- **The definition of a simulation.**
- **How to create a simulation.**
- **Ways to generate random numbers.**



# Section 4-7
# Counting

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# Key Concept

**In many probability problems, the big obstacle is finding the total number of outcomes, and this section presents several methods for finding such numbers without directly listing and counting the possibilities.**

# Fundamental Counting Rule

**For a sequence of two events in which the first event can occur $m$ ways and the second event can occur $n$ ways, the events together can occur a total of $m \cdot n$ ways.**

# Notation

The factorial symbol ! denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$

By special definition, 0! = 1.

# Factorial Rule

**A collection of *n* different items can be arranged in order *n*! different ways. (This factorial rule reflects the fact that the first item may be selected in *n* different ways, the second item may be selected in *n* – 1 ways, and so on.)**

# Permutations Rule
## (when items are all different)

**Requirements:**

1. **There are *n* different items available. (This rule does not apply if some of the items are identical to others.)**
2. **We select *r* of the *n* items (without replacement).**
3. **We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately.)**

**If the preceding requirements are satisfied, the number of permutations (or sequences) of *r* items selected from *n* available items (without replacement) is**

$$_nP_r = \frac{n!}{(n-r)!}$$

# Permutations Rule
## (when some items are identical to others)

**Requirements:**

1. There are $n$ items available, and some items are identical to others.
2. We select all of the $n$ items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are $n_1$ alike, $n_2$ alike, . . . $n_k$ alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \cdot n_2! \cdots\cdots n_k!}$$

# Combinations Rule

**Requirements:**

1. There are *n* different items available.
2. We select *r* of the *n* items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of ABC is the same as CBA.)

If the preceding requirements are satisfied, the number of combinations of *r* items selected from *n* different items is

$$_{n}C_{r} = \frac{n!}{(n - r)!\, r!}$$

# Permutations versus Combinations

**When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.**

# Recap

**In this section we have discussed:**

- **The fundamental counting rule.**
- **The factorial rule.**
- **The permutations rule (when items are all different).**
- **The permutations rule (when some items are identical to others).**
- **The combinations rule.**