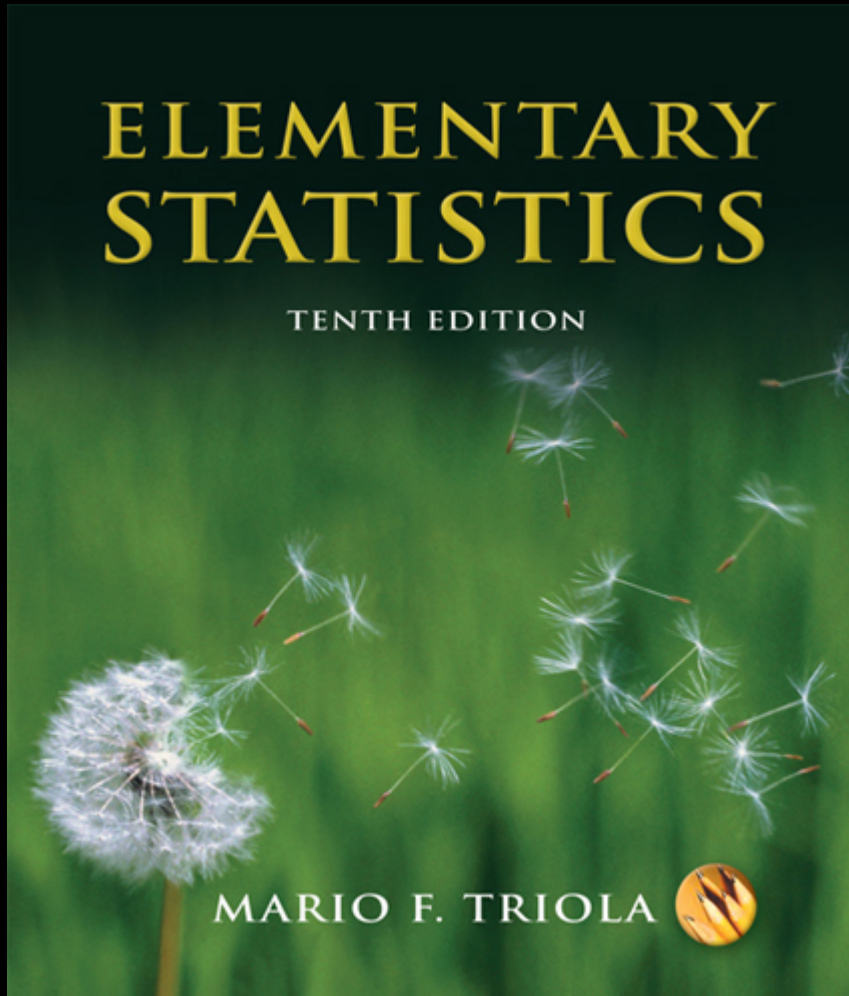


# Lecture Slides



## *Elementary Statistics* Tenth Edition

and the Triola Statistics Series

by Mario F. Triola

# Chapter 9

## Inferences from Two Samples

**9-1 Overview**

**9-2 Inferences About Two Proportions**

**9-3 Inferences About Two Means:  
Independent Samples**

**9-4 Inferences from Matched Pairs**

**9-5 Comparing Variation in Two Samples**

# Section 9-1 Overview



Created by Erin Hodgess, Houston, Texas  
Revised to accompany 10<sup>th</sup> Edition, Tom Wegleitner, Centreville, VA



# Overview

**There are many important and meaningful situations in which it becomes necessary to compare **two** sets of sample data.**

**This chapter extends the same methods introduced in Chapters 7 and 8 to situations involving two samples instead of only one.**



# **Section 9-2**

# **Inferences About Two**

# **Proportions**

Created by Erin Hodgess, Houston, Texas  
Revised to accompany 10<sup>th</sup> Edition, Tom Wegleitner, Centreville, VA



# Key Concept

**This section presents methods for using two sample proportions for constructing a confidence interval estimate of the difference between the corresponding population proportions, or testing a claim made about the two population proportions.**

# Requirements

1. We have proportions from two **independent** simple random samples.
2. For each of the two samples, the number of successes is at least 5 and the number of failures is at least 5.

# Notation for Two Proportions

For population 1, we let:

$p_1$  = **population** proportion

$n_1$  = size of the sample

$x_1$  = number of successes in the sample

$\hat{p}_1 = \frac{x_1}{n_1}$  (the **sample** proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

The corresponding meanings are attached to  $p_2$ ,  $n_2$ ,  $x_2$ ,  $\hat{p}_2$ , and  $\hat{q}_2$ , which come from population 2.



# Pooled Sample Proportion

- ❖ The **pooled sample proportion** is denoted by  $\bar{p}$  and is given by:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- ❖ We denote the complement of  $\bar{p}$  by  $\bar{q}$ ,  
so  $\bar{q} = 1 - \bar{p}$

# Test Statistic for Two Proportions

For  $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$ ,  $H_1: p_1 < p_2$ ,  $H_1: p_1 > p_2$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

# Test Statistic for Two Proportions - cont

For  $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$ ,  $H_1: p_1 < p_2$ ,  $H_1: p_1 > p_2$

where  $p_1 - p_2 = 0$  (assumed in the null hypothesis)

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{and} \quad \bar{q} = 1 - \bar{p}$$

# Test Statistic for Two Proportions - cont

**P-value:** Use Table A-2. (Use the computed value of the test statistic  $z$  and find its  $P$ -value by following the procedure summarized by Figure 8-6 in the text.)

**Critical values:** Use Table A-2. (Based on the significance level  $\alpha$ , find critical values by using the procedures introduced in Section 8-2 in the text.)

**Example:** For the sample data listed in the Table below, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Racial Profiling Data		
	Race and Ethnicity	
	Black and Non-Hispanic	White and Non-Hispanic
Drivers stopped by police	24	147
Total number of observed drivers	200	1400
<b>Percent Stopped by Police</b>	<b>12.0%</b>	<b>10.5%</b>

**Example:** For the sample data listed in the previous Table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

$$H_0: p_1 = p_2, H_1: p_1 > p_2$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{24 + 147}{200 + 1400} = 0.106875$$

$$\bar{q} = 1 - 0.106875 = 0.893125.$$

**Example:** For the sample data listed in the previous Table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$z = \frac{(0.120 - 0.105) - 0}{\sqrt{\frac{(0.106875)(0.893125)}{200} + \frac{(0.106875)(0.893125)}{1400}}}$$

$$z = 0.64$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

**Example:** For the sample data listed in the previous Table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

$$z = 0.64$$

This is a right-tailed test, so the  $P$ -value is the area to the right of the test statistic  $z = 0.64$ . The  $P$ -value is 0.2611.

Because the  $P$ -value of 0.2611 is greater than the significance level of  $\alpha = 0.05$ , we fail to reject the null hypothesis.



**Example:** For the sample data listed in the previous Table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$z = 0.64$$

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

Because we fail to reject the null hypothesis, we conclude that there is not sufficient evidence to support the claim that the proportion of black drivers stopped by police is greater than that for white drivers. This does **not** mean that racial profiling has been disproved. The evidence might be strong enough with more data.

**Example:** For the sample data listed in the previous Table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$n_1 = 200$$

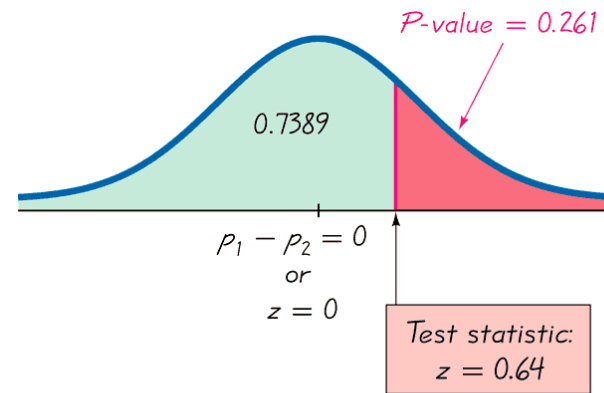
$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

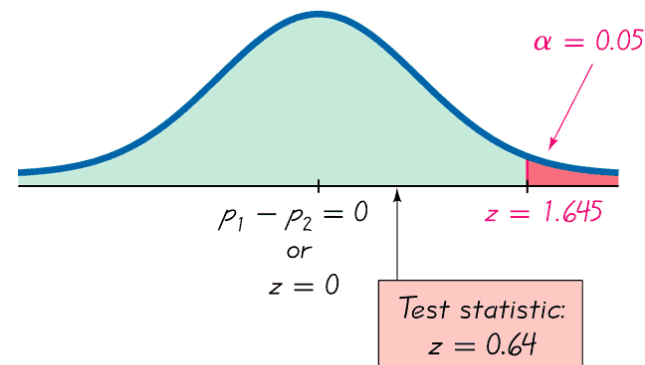
$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$



(a) P-Value Method



(b) Traditional Method

# Confidence Interval Estimate of $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

where  $E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$

**Example:** For the sample data listed in the previous Table, find a 90% confidence interval estimate of the difference between the two population proportions.

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

$$E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$E = 1.645 \sqrt{\frac{(.12)(.88)}{200} + \frac{(0.105)(0.895)}{1400}}$$

$$E = 0.040$$

**Example:** For the sample data listed in the previous table, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

$$n_1 = 200$$

$$x_1 = 24$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$$

$$(0.120 - 0.105) - 0.040 < (p_1 - p_2) < (0.120 - 0.105) + 0.040 - 0.025 < (p_1 - p_2) < 0.055$$

$$n_2 = 1400$$

$$x_2 = 147$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$$

# Why Do the Procedures of This Section Work?

**The text contains a detailed explanation of how and why the test statistic given for hypothesis tests is justified. Be sure to study it carefully.**

# Recap

**In this section we have discussed:**

- ❖ **Requirements for inferences about two proportions.**
- ❖ **Notation.**
- ❖ **Pooled sample proportion.**
- ❖ **Hypothesis tests.**



# **Section 9-3**

## **Inferences About Two Means: Independent Samples**

Created by Erin Hodgess, Houston, Texas  
Revised to accompany 10<sup>th</sup> Edition, Tom Wegleitner, Centreville, VA





# Key Concept

**This section presents methods for using sample data from two independent samples to test hypotheses made about two population means or to construct confidence interval estimates of the difference between two population means.**

# Part 1: Independent Samples with $\sigma_1$ and $\sigma_2$ Unknown and Not Assumed Equal

# Definitions

Two samples are **independent** if the sample values selected from one population are not related to or somehow paired or matched with the sample values selected from the other population.

Two samples are **dependent** (or consist of **matched pairs**) if the members of one sample can be used to determine the members of the other sample.

# Requirements

1.  $\sigma_1$  and  $\sigma_2$  are unknown and no assumption is made about the equality of  $\sigma_1$  and  $\sigma_2$ .
2. The two samples are **independent**.
3. Both samples are **simple random samples**.
4. Either or both of these conditions are satisfied: The two sample sizes are both **large** (with  $n_1 > 30$  and  $n_2 > 30$ ) or both samples come from populations having normal distributions.

# Hypothesis Test for Two Means: Independent Samples

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

# Hypothesis Test - cont

## Test Statistic for Two Means: Independent Samples

- Degrees of freedom:** In this book we use this simple and conservative estimate:  
 $df = \text{smaller of } n_1 - 1 \text{ and } n_2 - 1.$
- P-values:** Refer to Table A-3. Use the procedure summarized in Figure 8-6.
- Critical values:** Refer to Table A-3.

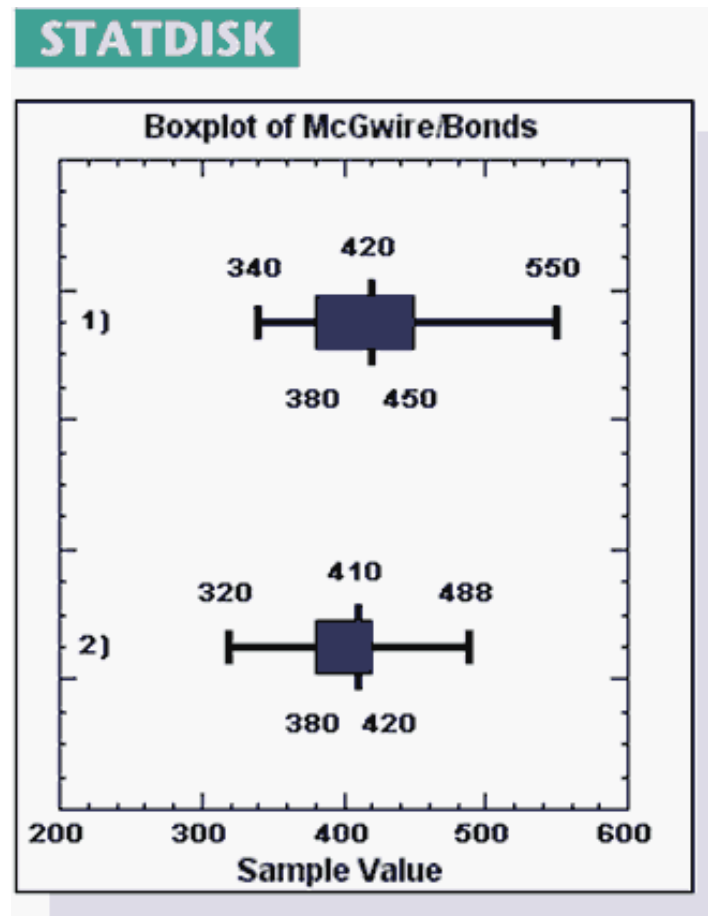
# McGwire Versus Bonds

Sample statistics are shown for the distances of the home runs hit in record-setting seasons by Mark McGwire and Barry Bonds. Use a 0.05 significance level to test the claim that the distances come from populations with different means.

	McGwire	Bonds
$n$	70	73
$\bar{x}$	418.5	403.7
$s$	45.5	30.6

# McGwire Versus Bonds - cont

Below is a Statdisk plot of the data





# McGwire Versus Bonds - cont

**Claim:**  $\mu_1 \neq \mu_2$

**$H_o$  :**  $\mu_1 = \mu_2$

**$H_1$  :**  $\mu_1 \neq \mu_2$

**$\alpha = 0.05$**

**$n_1 - 1 = 69$**

**$n_2 - 1 = 72$**

**df = 69**

**$t_{.025} = 1.994$**

# McGwire Versus Bonds - cont

## Test Statistic for Two Means:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

# McGwire Versus Bonds - cont

## Test Statistic for Two Means:

$$t = \frac{(418.5 - 403.7) - 0}{\sqrt{\frac{45.5^2}{70} + \frac{30.6^2}{73}}}$$
$$= 2.273$$

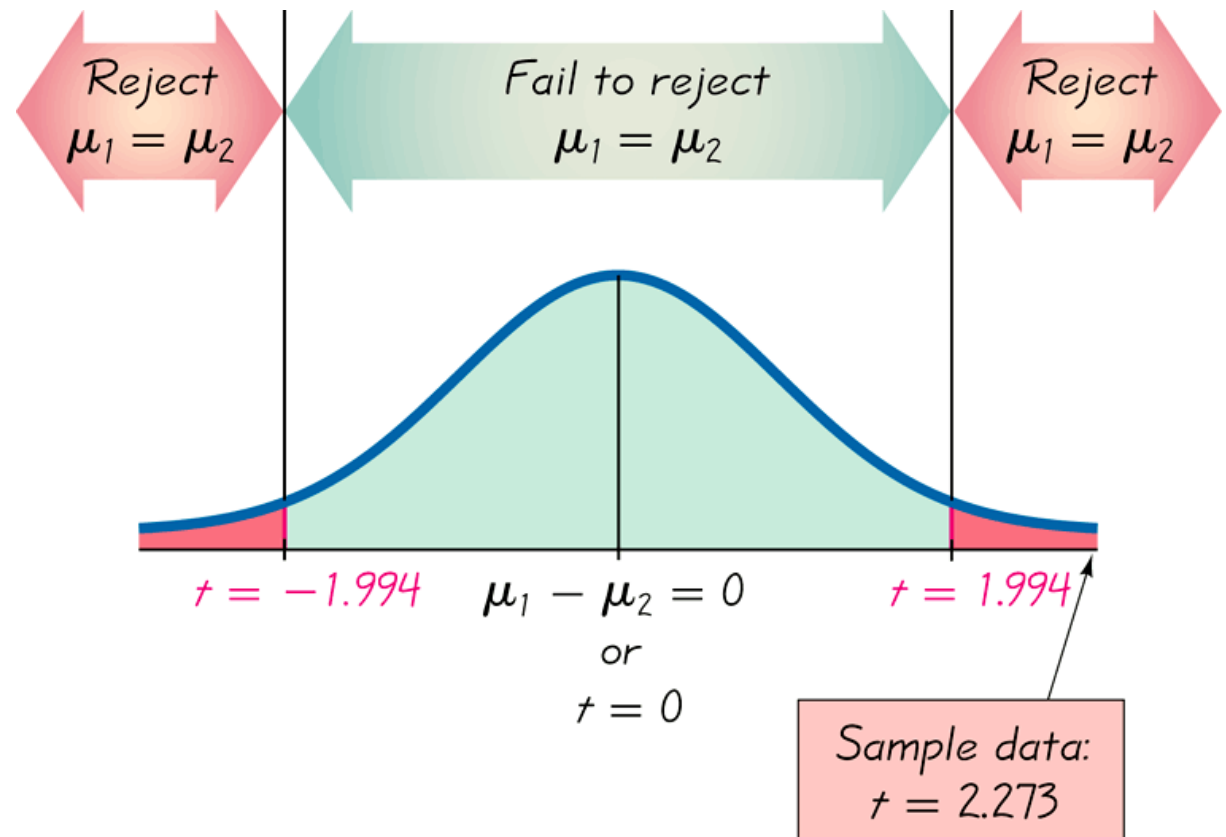
# McGwire Versus Bonds - cont

Claim:  $\mu_1 \neq \mu_2$

$H_o : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\alpha = 0.05$



# McGwire Versus Bonds - cont

**Claim:**  $\mu_1 \neq \mu_2$

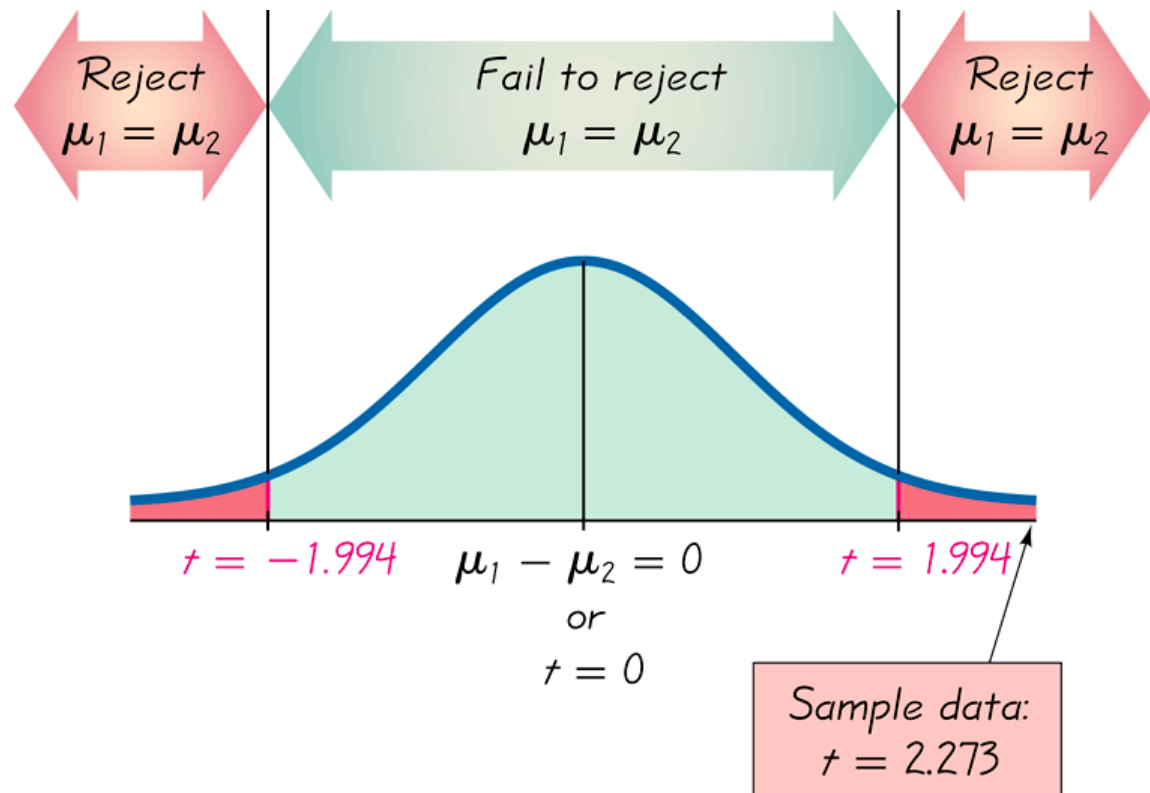
$H_o : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\alpha = 0.05$

There is significant evidence to support the claim that there is a difference between the mean home run distances of Mark McGwire and Barry Bonds.

**Reject the  
Null  
Hypothesis**



# Confidence Intervals

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where  $E = z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

# McGwire Versus Bonds

## Confidence Interval Method

Using the data given in the preceding example, construct a 95% confidence interval estimate of the difference between the mean home run distances of Mark McGwire and Barry Bonds.

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$
$$E = 1.994 \sqrt{\frac{45.5^2}{70} + \frac{30.6^2}{73}}$$
$$E = 13.0$$

# McGwire Versus Bonds

## Confidence Interval Method - cont

Using the data given in the preceding example, construct a 95% confidence interval estimate of the difference between the mean home run distances of Mark McGwire and Barry Bonds.

$$(418.5 - 403.7) - 13.0 < (\mu_1 - \mu_2) < (418.5 - 403.7) + 13.0$$
$$1.8 < (\mu_1 - \mu_2) < 27.8$$

**We are 95% confident that the limits of 1.8 ft and 27.8 ft actually do contain the difference between the two population means.**



# Part 2: Alternative Methods

# Independent Samples with $\sigma_1$ and $\sigma_2$ Known.

# Requirements

1. The two population standard deviations are both known.
2. The two samples are **independent**.
3. Both samples are **simple random samples**.
4. Either or both of these conditions are satisfied: The two sample sizes are both **large** (with  $n_1 > 30$  and  $n_2 > 30$ ) or both samples come from populations having normal distributions.

# Hypothesis Test for Two Means: Independent Samples with $\sigma_1$ and $\sigma_2$ Both Known

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

**P-values and critical values:** Refer to Table A-2.

# Confidence Interval: Independent Samples with $\sigma_1$ and $\sigma_2$ Both Known

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where  $E = z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

# Methods for Inferences About Two Independent Means

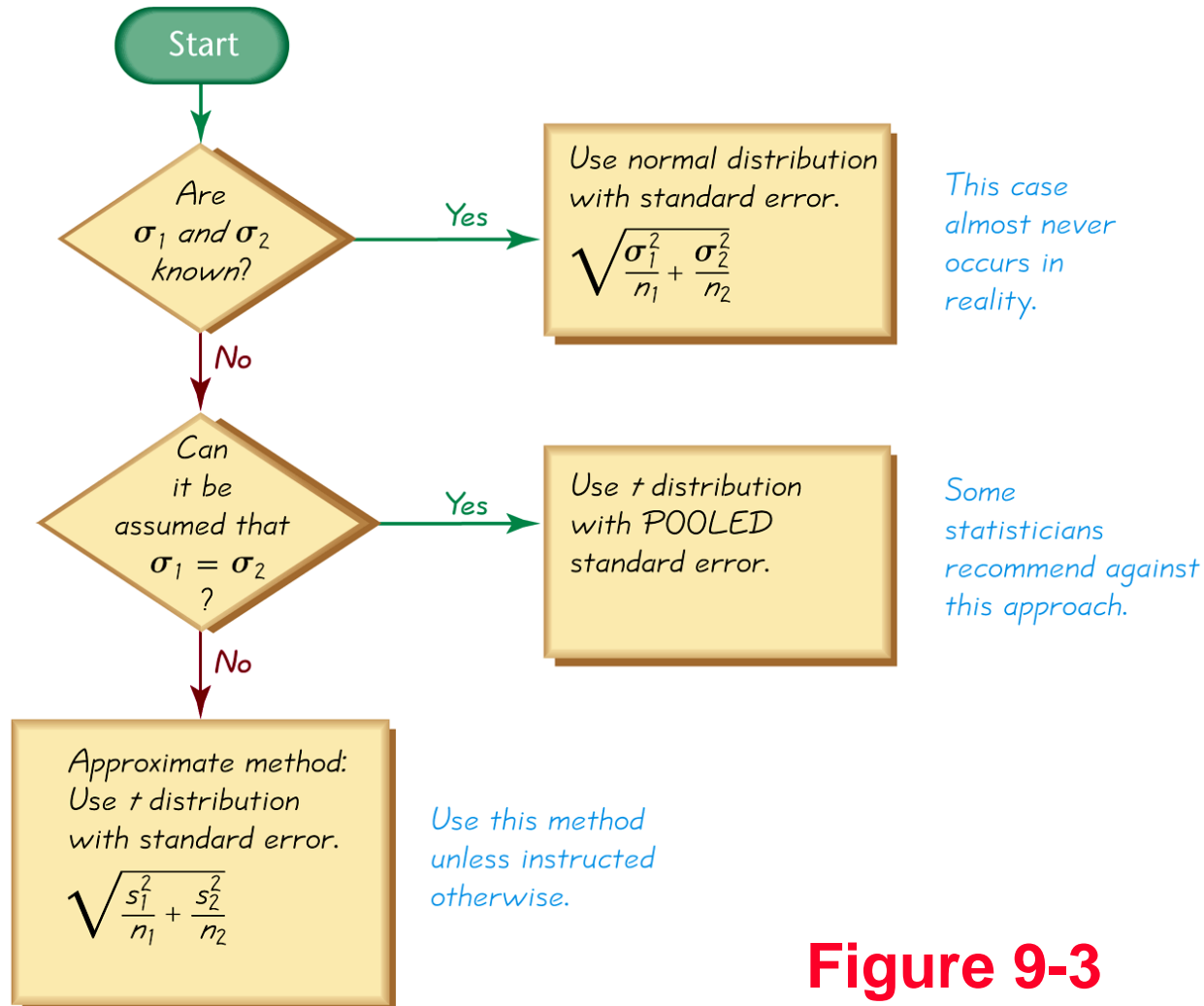


Figure 9-3

Assume that  $\sigma_1 = \sigma_2$  and **Pool** the  
Sample Variances.

# Requirements

1. The two population standard deviations are not known, but they are assumed to be equal. That is  $\sigma_1 = \sigma_2$ .
2. The two samples are **independent**.
3. Both samples are **simple random samples**.
4. Either or both of these conditions are satisfied: The two sample sizes are both **large** (with  $n_1 > 30$  and  $n_2 > 30$ ) or both samples come from populations having normal distributions.



# Hypothesis Test Statistic for Two Means: Independent Samples and

$$\sigma_1 = \sigma_2$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Where

$$s_p^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

and the number of degrees of freedom is  $df = n_1 + n_2 - 2$

# Confidence Interval Estimate of $\mu_1 - \mu_2$ : Independent Samples with $\sigma_1 = \sigma_2$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

$$\text{where } E = t_{\alpha/2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

and number of degrees of freedom is  $df = n_1 + n_2 - 2$

# Strategy

**Unless instructed otherwise, use the following strategy:**

**Assume that  $\sigma_1$  and  $\sigma_2$  are unknown, do **not** assume that  $\sigma_1 = \sigma_2$ , and use the test statistic and confidence interval given in Part 1 of this section. (See Figure 9-3.)**

# Recap

**In this section we have discussed:**

- ❖ **Independent samples with the standard deviations unknown and not assumed equal.**
- ❖ **Alternative method where standard deviations are known**
- ❖ **Alternative method where standard deviations are assumed equal and sample variances are pooled.**



# **Section 9-4**

## **Inferences from Matched Pairs**

Created by Erin Hodgess, Houston, Texas  
Revised to accompany 10<sup>th</sup> Edition, Tom Wegleitner, Centreville, VA



# Key Concept

**In this section we develop methods for testing claims about the mean difference of matched pairs.**

**For each matched pair of sample values, we find the difference between the two values, then we use those sample differences to test claims about the population difference or to construct confidence interval estimates of the population difference.**

# Requirements

1. The sample data consist of matched pairs.
2. The samples are simple random samples.
3. Either or both of these conditions is satisfied: The number of matched pairs of sample data is large ( $n > 30$ ) or the pairs of values have differences that are from a population having a distribution that is approximately normal.

# Notation for Matched Pairs

- $d$  = individual difference between the two values of a single matched pair
- $\mu_d$  = mean value of the differences  $d$  for the **population** of paired data
- $\bar{d}$  = mean value of the differences  $d$  for the paired **sample** data (equal to the mean of the  $x - y$  values)
- $S_d$  = standard deviation of the differences  $d$  for the paired **sample** data
- $n$  = number of **pairs** of data.



# Hypothesis Test Statistic for Matched Pairs

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

where degrees of freedom =  $n - 1$

# ***P*-values and Critical Values**

**Use Table A-3 (*t*-distribution).**

# Confidence Intervals for Matched Pairs

$$\bar{d} - E < \mu_d < \bar{d} + E$$

$$\text{where } E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

**Critical values of  $t_{\alpha/2}$  : Use Table A-3 with  $n - 1$  degrees of freedom.**

# Are Forecast Temperatures Accurate?

**The following Table consists of five actual low temperatures and the corresponding low temperatures that were predicted five days earlier. The data consist of matched pairs, because each pair of values represents the same day. Use a 0.05 significance level to test the claim that there is a difference between the actual low temperatures and the low temperatures that were forecast five days earlier.**

# Are Forecast Temperatures Accurate? - cont

Actual and Forecast Temperature					
Actual low	1	−5	−5	23	9
Low forecast five days earlier	16	16	20	22	15
Difference $d = \text{actual} - \text{predicted}$	−15	−21	−25	1	−6

# Are Forecast Temperatures Accurate? - cont

$$\bar{d} = -13.2$$

$$s = 10.7$$

$$n = 5$$

$$t_{\alpha/2} = 2.776 \text{ (found from Table A-3 with 4 degrees of freedom and 0.05 in two tails)}$$

# Are Forecast Temperatures Accurate? - cont

$$H_0: \mu_d = 0$$

$$H_1: \mu_d \neq 0$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{-13.2 - 0}{\frac{10.7}{\sqrt{5}}} = -2.759$$

# Are Forecast Temperatures Accurate? - cont

$$H_0: \mu_d = 0$$

$$H_1: \mu_d \neq 0$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{-13.2 - 0}{\frac{10.7}{\sqrt{5}}} = -2.759$$

Because the test statistic does not fall in the critical region, we fail to reject the null hypothesis.



# Are Forecast Temperatures Accurate? - cont

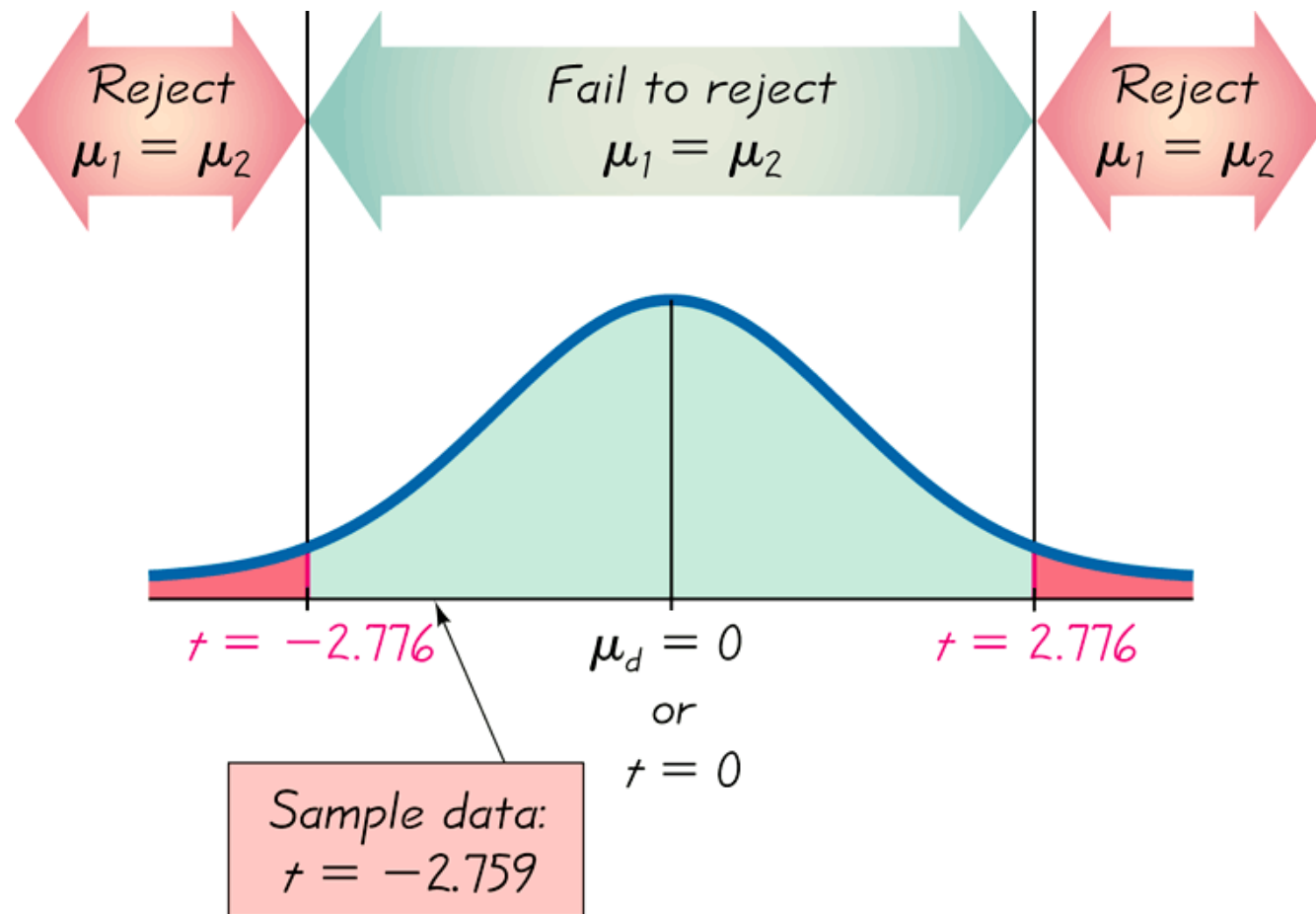
$$H_0: \mu_d = 0$$

$$H_1: \mu_d \neq 0$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{-13.2 - 0}{\frac{10.7}{\sqrt{5}}} = -2.759$$

The sample data in the previous Table do not provide sufficient evidence to support the claim that actual and five-day forecast low temperatures are different.

# Are Forecast Temperatures Accurate? - cont



# Are Forecast Temperatures Accurate? - cont

**Using the same sample matched pairs in the previous Table, construct a 95% confidence interval estimate of  $\mu_d$ , which is the mean of the differences between actual low temperatures and five-day forecasts.**

# Are Forecast Temperatures Accurate? - cont

$$E = t_{\alpha/2} \frac{S_d}{\sqrt{n}}$$

$$E = (2.776) \left( \frac{10.7}{\sqrt{5}} \right)$$

$$= 13.3$$

# Are Forecast Temperatures Accurate? - cont

$$\begin{aligned}\bar{d} - E &< \mu_d < \bar{d} + E \\ -13.2 - 13.3 &< \mu_d < -13.2 + 13.3 \\ -26.5 &< \mu_d < 0.1\end{aligned}$$

# **Are Forecast Temperatures Accurate? - cont**

**In the long run, 95% of such samples will lead to confidence intervals that actually do contain the true population mean of the differences.**

# Recap

**In this section we have discussed:**

- ❖ **Requirements for inferences from matched pairs.**
- ❖ **Notation.**
- ❖ **Hypothesis test.**
- ❖ **Confidence intervals.**



# **Section 9-5**

# **Comparing Variation in**

# **Two Samples**

Created by Erin Hodgess, Houston, Texas  
Revised to accompany 10<sup>th</sup> Edition, Tom Wegleitner, Centreville, VA





# Key Concept

**This section presents the  $F$  test for using two samples to compare two population variances (or standard deviations). We introduce the  $F$  distribution that is used for the  $F$  test.**

**Note that the  $F$  test is **very** sensitive to departures from normal distributions.**

# Measures of Variation

$s$  = standard deviation of **sample**

$\sigma$  = standard deviation of **population**

$s^2$  = variance of **sample**

$\sigma^2$  = variance of **population**

# Requirements

1. The two populations are **independent** of each other.
2. The two populations are each **normally distributed**.

# Notation for Hypothesis Tests with Two Variances or Standard Deviations

$s_1^2$  = **larger** of the two sample variances

$n_1$  = size of the sample with the **larger** variance

$\sigma_1^2$  = variance of the population from which the sample with the **larger** variance was drawn

The symbols  $s_2^2$ ,  $n_2$ , and  $\sigma_2^2$  are used for the other sample and population.

# Test Statistic for Hypothesis Tests with Two Variances

$$F = \frac{s_1^2}{s_2^2}$$

Where  $s_1^2$  is the larger of the two sample variances

**Critical Values:** Using Table A-5, we obtain critical  $F$  values that are determined by the following three values:

1. The significance level  $\alpha$
2. Numerator degrees of freedom =  $n_1 - 1$
3. Denominator degrees of freedom =  $n_2 - 1$

# Properties of the $F$ Distribution

- ❖ The  $F$  distribution is not symmetric.
- ❖ Values of the  $F$  distribution cannot be negative.
- ❖ The exact shape of the  $F$  distribution depends on two different degrees of freedom.

# Properties of the $F$ Distribution - cont

If the two populations do have **equal variances**, then  $F = \frac{s_1^2}{s_2^2}$  will be close to 1 because  $s_1^2$  and  $s_2^2$  are close in value.

# Properties of the $F$ Distribution

## - cont

If the two populations have radically **different variances**, then  $F$  will be a large number.

Remember, the larger sample variance will be  $s_1^2$ .



# Conclusions from the $F$ Distribution

Consequently, a **value of  $F$  near 1** will be evidence **in favor** of the conclusion that  $\sigma_1^2 = \sigma_2^2$ .

But a **large value of  $F$**  will be evidence **against** the conclusion of equality of the population variances.

# Coke Versus Pepsi

Data Set 12 in Appendix B includes the weights (in pounds) of samples of regular Coke and regular Pepsi. Sample statistics are shown. Use the 0.05 significance level to test the claim that the weights of regular Coke and the weights of regular Pepsi have the same standard deviation.

	Regular Coke	Regular Pepsi
$n$	36	36
$\bar{x}$	0.81682	0.82410
$s$	0.007507	0.005701

# Coke Versus Pepsi

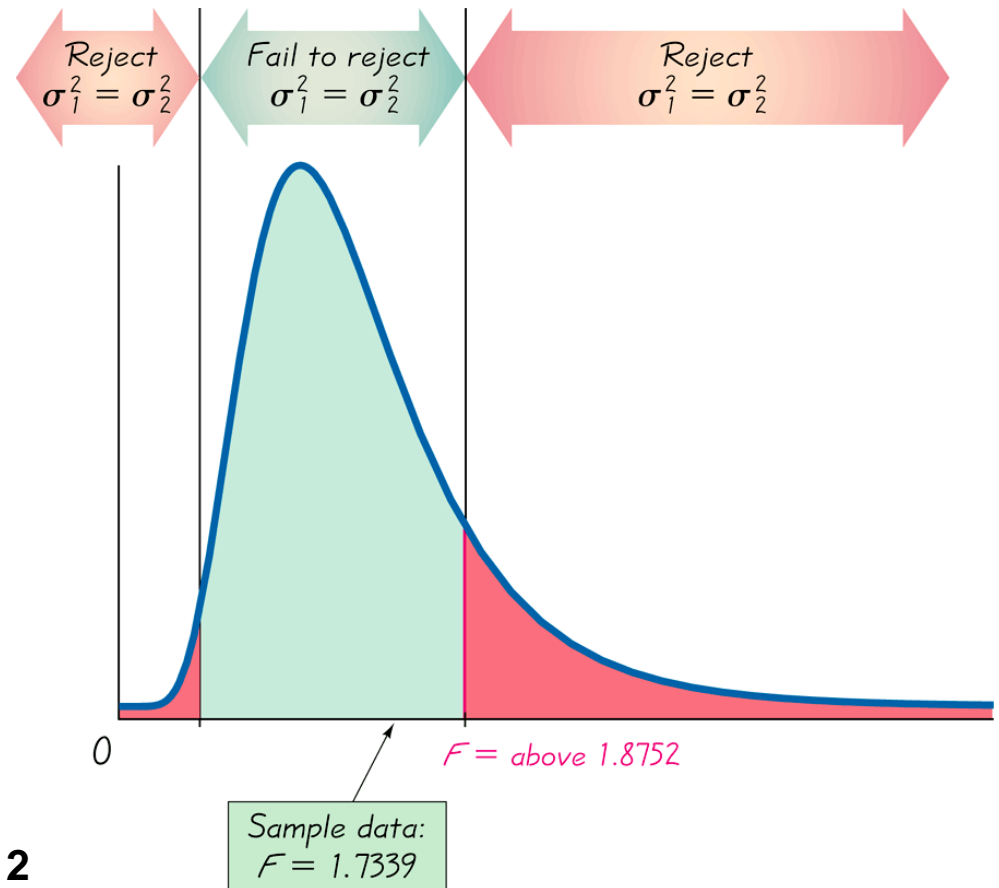
Claim:  $\sigma_1^2 = \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05$

$$\begin{aligned}\text{Value of } F &= \frac{s_1^2}{s_2^2} \\ &= \frac{0.007507^2}{0.005701^2} \\ &= \mathbf{1.7339}\end{aligned}$$



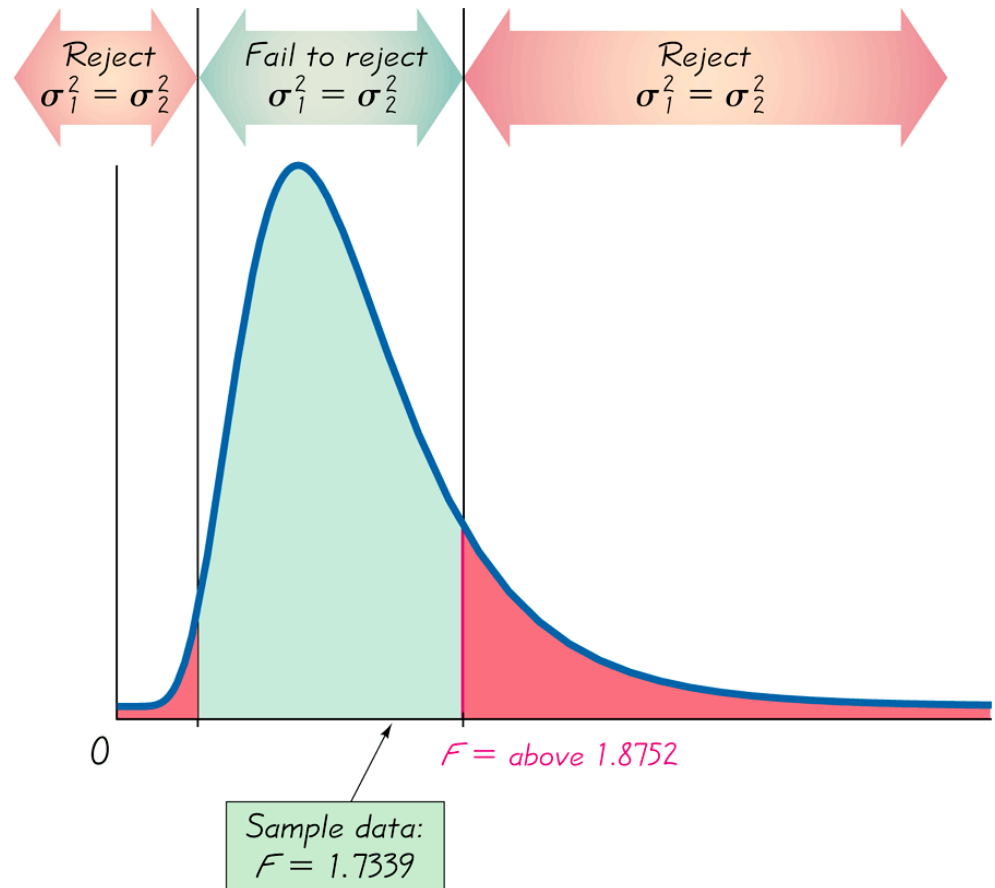
# Coke Versus Pepsi

**Claim:**  $\sigma_1^2 = \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05$



**There is not sufficient evidence to warrant rejection of the claim that the two variances are equal.**

# Recap

**In this section we have discussed:**

- ❖ **Requirements for comparing variation in two samples**
- ❖ **Notation.**
- ❖ **Hypothesis test.**
- ❖ **Confidence intervals.**
- ❖  **$F$  test and distribution.**