This problem comes from MATH 2414 Calculus II – David Katz

Compute the length of the curve $y = \sqrt{x - x^2} + Sin^{-1}(\sqrt{x})$ from $x = 0$ to $x = 1$.

The formula for length of a curve requires one to compute *y'*:

$$\frac{dy}{dx} = \frac{1}{2}\left(x - x^2\right)^{-\frac{1}{2}} * (1-2x) + \frac{1}{2\sqrt{x}\sqrt{1-\left(\sqrt{x}\right)^2}} \quad \text{(use the chain rule)}$$

$$= \frac{1-2x}{2\sqrt{x-x^2}} + \frac{1}{2\sqrt{x-x^2}} = \frac{2-2x}{2\sqrt{x-x^2}} = \frac{2(1-x)}{2\sqrt{x-x^2}} = \frac{\sqrt{(1-x)^2}}{\sqrt{x-x^2}} = \frac{\sqrt{(1-x)^2}}{\sqrt{x(1-x)}} = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}}$$

Putting all the parts together:

$$\text{Length} = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+\left(\sqrt{\frac{1-x}{x}}\right)^2}\,dx = \int_0^1 \sqrt{1+\frac{1-x}{x}}dx = \int_0^1 \sqrt{\frac{x}{x}+\frac{1-x}{x}}dx$$

$$= \int_0^1 \sqrt{\frac{1}{x}}dx = 2\sqrt{x}\,\Big|_0^1 = 2$$