This problem comes from MATH 2414 Calculus II – David Katz

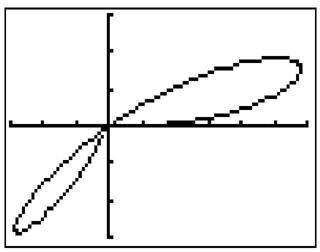
Compute the area inside the petals of the curve $r = 1 + 5\sin(6\theta)$.

Observe that the period of this curve is π/3 (= 2π/6). So starting at the point (r,θ)=(1,0), the curve draws 2 petals for each π/3 radians. As we move through the next period of the curve and draw additional petals, how will we know when we have drawn the complete picture will all possible petals?

There is no one answer to that question. But here are some guidelines to make sure we have a complete picture: rotate the smallest multiple of π which takes us through at least one period, and/or rotate until the curve returns to its starting point.

In this problem, we can stop rotating after 2π radians. At this point, the curve is at (1, 2π) which is the same as our starting point (1,0). Note that we completed 6 periods within the 2π radians: 6 = 2π ÷ π/3. Thus there are 12 total petals for this curve: 12 = 6 × 2.

The first period of the curve traces out 1 bigger petal and 1 smaller petal as illustrated in the screen capture below:



As we continue to trace out the remaining 10 petals, we notice that no petal is contained in whole or in part in another petal.

To apply the formula $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$ for area inside a polar curve, we integrate from α=0 to β=2π because all 12 petals are traced out in 2π radians without any petal overlapping another petal.

(If parts of the curve did overlap, we would have to integrate petal by petal to avoid over-counting the area.)

The WolframAlpha website computes this area to be just over 42 square units:

$$\int_0^{2\pi} \frac{1}{2}\left(1+5\sin(6\theta)\right)^2 d\theta = \frac{1}{48}\left(324\theta - 25\sin(12\theta) - 40\cos(6\theta)\right)\Big|_0^{2\pi} = \frac{27\pi}{2} \approx 42.4115$$