This problem comes from MATH 2412 Precalculus – David Katz

Is the partial fraction decomposition of a rational function unique?

Precalculus textbooks usually do not discuss this issue directly, but the answer is yes! The partial fraction decomposition for a rational function $p(x)/q(x)$ where $p(x)$ and $q(x)$ are polynomials always exists and is unique, except for rearrangement of terms or multiplication by a constant. So if two students are computing the same partial fraction decomposition, and assuming they both do their math correctly, their answers must match exactly.

A proof of this concept using the mathematics commonly studied at the community college can be found in the article *Two Proofs of the Existence and Uniqueness of the Partial Fraction Decomposition* by Bradley and Cook at http://www.m-hikari.com/forth/cookIMF29-32-2012.pdf.

Not all partial fraction decompositions have to be unique. For example, the partial fraction decomposition of a fraction may not be unique:

$$\frac{5}{12} = \frac{2}{3} + \frac{-1}{2} + \frac{1}{2^2} = \frac{-1}{3} + \frac{0}{2} + \frac{3}{2^2}$$

But for the problems one does in precalculus with rational functions, the partial fraction decompositions are always unique.