This problem comes from MATH 1314 College Algebra – David Katz

Can all polynomial equations be solved with the Rational Zeros Theorem or the quadratic formula?

Yes and no. The authors of College Algebra textbooks are usually nice enough to reward diligent students with at least one easy-to-find solution or zero through use of the Rational Zeros Theorem or the quadratic formula.

However, in general, one can create polynomials whose solutions cannot be computed through the Rational Zeros Theorem or the quadratic formula.

Below are three such polynomial equations:

(1) $x^3 - 2x^2 + 8x + 50 = 0$

(2) $-6x^3 - x^2 + x + 10 = 0$

(3) $x^3 + x\sqrt{2} + 10 = 0$

Of course, these polynomials must have at least one real solution (why?). None of the possible rational solutions from the Rational Zeros Theorem work in polynomials (1) and (2). And the Rational Zeros Theorem does not even apply in polynomial (3) because it has a non-integer coefficient. Что делать? or what is to be done? One must resort to Cardano's (16^{th} century) cubic formula for solving cubic polynomials.

Interestingly, Cardano's formula is about two pages long, quite a contrast with the relatively simple one-line quadratic formula. So solving the two polynomial equations above will take some time, but the solutions can be found. The solutions will involve radicals within radicals so the answers will look quite different from the usual answers a student encounters in College Algebra class.