This problem comes from MATH 2413 Calculus I – David Katz

All real and complex numbers (besides 0) have three distinct cube roots. For example, if you compute the (-8)^(1/3) on your TI calculator your answer is displayed as -2. But if you compute the cube root of -8 on *Mathematica*, the answer is $1+i\sqrt{3}$.

Why the difference?

Mathematica and its related website Wolfram Alpha (www.wolframalpha.com) are programmed to compute the principal cube root of a number. To understand which of the three roots of a cube is taken as the principal root, express the three cube roots in polar form using non-negative r and $-\pi < \theta \le \pi$.

> The root with the angle θ that is closest to or on the polar axis is called the principal root. If two angles are equidistant from the polar axis, the positive angle is chosen.

The polar form of the cube roots of -8 are $(2 \text{ cis } \pi)$, $(2 \text{ cis } \pi/3)$, and $(2 \text{ cis } -\pi/3)$. The last two roots are equidistant from the polar axis, but $\pi/3 > 0$, so $(2 \text{ cis } \pi/3)$ is the principal root. The $a+bi$ form of $(2 \text{ cis } \pi/3)$ is $1+i\sqrt{3}$.

Another Example

The polar form of the cube roots of $1 - i$ are:

$$\left(\sqrt[6]{2} \text{ cis } \frac{7\pi}{12} \right), \left(\sqrt[6]{2} \text{ cis } \frac{-9\pi}{12} \right), \text{ and } \left(\sqrt[6]{2} \text{ cis } \frac{-\pi}{12} \right)$$

The angle $-\pi/24$ is closest to the polar axis, so the principal cube root of $1 - i$ is:

$$\left(\sqrt[6]{2} \text{ cis } \frac{-\pi}{12} \right)$$