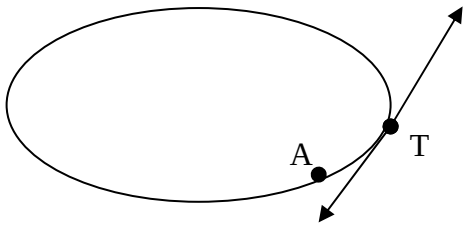
This problem comes from MATH 2412 Precalculus – David Katz

Given the picture of some polynomial curve, draw the line tangent on that picture at a specified point.

Consider this picture of a curve and a tangent line at point T.



The most common way to draw a tangent line is to consider the tangent line as the limit of secant lines approaching the point of tangency T. In this approach, position point A on the curve as close as you can next to point T. Draw the secant line thru points A and T. This secant line is your best approximation for the tangent line at point T.

But this approach is but a (sorry – pun alert) *tangent* to the rich history of constructing the tangent lines. Is there a method to constructing tangent lines which does not use notions of limits or derivatives?

You can construct tangents to circles easy enough. A tangent line only requires one to construct a perpendicular to a radius at a given point of tangency on the circle.

You can also construct tangents to the other conic sections (ellipse, hyperbola, parabola) by applying the techniques of coordinate geometry to properties of symmetry and reflection for these curves. Fuller and Tarwater took this approach in their famous textbook *Analytic Geometry.* Larson's *Precalculus With Limits* textbook uses a similar method. Techniques are also available for constructing tangents to conic sections by straight edge and compass alone.

For tangents to other polynomial curves, Descartes in the second book of his *Geometrie* and Fermat in his posthumous *Methodus* discussed ways to move a secant line so that the limit of this movement converges to the actual tangent line. Ultimately, these techniques rely on some notion of limits. A good overview of this topic can be found in H. Thurston's paper (1964) *On the Definition of a Tangent Line*.

Of course today, the much easier and more straightforward computation of derivatives replaces the need for these various *ad hoc* methods for determining a tangent line.

Interested in seeing any of these tangent line techniques in action? Contact me, and I can put on a demonstration for you.