To find the quartic regression polynomial $y=p(x)$ that best fits or matches a set of n data points (x_i,y_i), we need to minimize the difference between the computed value of y and the actual value of y_i for each data point.

In other words we need to minimize this function:

$$S(a,b,c,d,e)=\sum_{i=1}^{n}\left(ax_i^4+bx_i^3+cx_i^2+dx_i+e-y_i\right)^2$$

The function S is a function of five variables $\{a,b,c,d,e\}$, which represent the five coefficients for the quartic polynomial $p(x)$.

- Note: we assume $n>5$ distinct data points; if not, there is no need to minimize S, and one can easily find a quartic polynomial that perfectly matches the data (i.e., $r = 1$).

- Note: in Calculus, one learns that the minimization of S is found by setting each of the 5 partial derivatives equal to zero. In a course on Linear Algebra, one learns that the resulting system of equations has one solution, and that the solution contains the optimal values for the coefficients $\{a,b,c,d,e\}$ of the quartic regression equation $y=p(x)$.

The system of equations we need to solve to find the coefficients $\{a,b,c,d,e\}$ of the regression polynomial $y=p(x)$ is:

$$2\sum_{i=1}^{n}\left(ax_i^8+bx_i^7+cx_i^6+dx_i^5+ex_i^4-2x_i^4y_i\right)=0$$

$$2\sum_{i=1}^{n}\left(ax_i^7+bx_i^6+cx_i^5+dx_i^4+ex_i^3-2x_i^3y_i\right)=0$$

$$2\sum_{i=1}^{n}\left(ax_i^6+bx_i^5+cx_i^4+dx_i^3+ex_i^2-2x_i^2y_i\right)=0$$

$$2\sum_{i=1}^{n}\left(ax_i^5+bx_i^4+cx_i^3+dx_i^2+ex_i^1-2x_i^1y_i\right)=0$$

$$2\sum_{i=1}^{n}\left(ax_i^4+bx_i^3+cx_i^2+dx_i^1+ex_i^0-2x_i^0y_i\right)=0$$

This system can be re-expressed as an augmented matrix:

$$2\sum_{i=1}^{n}\left(\begin{array}{ccccc|c} x_i^8 & x_i^7 & x_i^6 & x_i^5 & x_i^4 & x_i^4 y_i \\ x_i^7 & x_i^6 & x_i^5 & x_i^4 & x_i^3 & x_i^3 y_i \\ x_i^6 & x_i^5 & x_i^4 & x_i^3 & x_i^2 & x_i^2 y_i \\ x_i^5 & x_i^4 & x_i^3 & x_i^2 & x_i & x_i y_i \\ x_i^4 & x_i^3 & x_i^2 & x_i & 1 & y_i \end{array}\right)$$

Observe that this augmented matrix requires the computation of some relatively high powers of x_i. When a computer/calculator attempts to solve this system through some type of numerical process like Gauss-Jordan reduction, these computations may rapidly exceed the capability of the computer/calculator.