Daniel Bernoulli

Born: 8 Feb 1700 in Groningen, Netherlands
Died: 17 March 1782 in Basel, Switzerland

Daniel Bernoulli was the son of Johann Bernoulli. He was born in Groningen while his father held the chair of mathematics there. His older brother was Nicolaus(II) Bernoulli and his uncle was Jacob Bernoulli so he was born into a family of leading mathematicians but also into a family where there was unfortunate rivalry, jealousy and bitterness.

When Daniel was five years old the family returned to their native city of Basel where Daniel's father filled the chair of mathematics left vacant on the death of his uncle Jacob Bernoulli. When Daniel was five years old his younger brother Johann(II) Bernoulli was born. All three sons would go on to study mathematics but this was not the course that Johann Bernoulli planned for Daniel.

Johann Bernoulli's father had tried to force Johann into a business career and he had resisted strongly. Rather strangely Johann Bernoulli now tried exactly the same with his own son Daniel. First however Daniel was sent to Basel University at the age of 13 to study philosophy and logic. He obtained his baccalaureate examinations in 1715 and went on to obtain his master's degree in 1716. Daniel, like his father, really wanted to study mathematics and during the time he studied philosophy at Basel, he was learning the methods of the calculus from his father and his older brother Nicolaus(II) Bernoulli.

Johann was determined that Daniel should become a merchant and he tried to place him in an apprenticeship. However Daniel was as strongly opposed to this as his own father had been and soon Johann relented but certainly not as far as to let Daniel study mathematics. Johann declared that there was no money in mathematics and so he sent Daniel back to Basel University to study medicine. This Daniel did spending time studying medicine at Heidelberg in 1718 and Strasbourg in 1719. He returned to Basel in 1720 to complete his doctorate in medicine.

By this stage Johann Bernoulli was prepared to teach his son more mathematics while he studied medicine and Daniel studied his father's theories of kinetic energy. What he learned on the conservation of energy from his father he applied to his medical studies and Daniel wrote his doctoral dissertation on the mechanics of breathing. So like his father Daniel had applied mathematical physics to medicine in order to obtain his medical doctorate.

Daniel wanted to embark on an academic career like his father so he applied for two chairs at Basel. His application for the chair of anatomy and botany was decided by drawing of lots and he was unlucky in this game of chance. The next chair to fall vacant at Basel that Daniel applied for was the chair of logic, but again the game of chance of the final selection by drawing of lots went against him. Having failed to obtain an academic post, Daniel went to Venice to study practical medicine.

In Venice Daniel was severely ill and so was unable to carry out his intention of travelling to Padua to further his medical studies. However, while in Venice he worked on mathematics and his first mathematical work was published in 1724 when, with Goldbach's assistance, *Mathematical exercises* was published. This consisted of four separate parts being four topics that had attracted his interest while in Venice.

The first part described the game of faro and is of little importance other than showing that Daniel was learning about probability at this time. The second part was on the flow of water from a hole in a container and discussed Newton's theories (which were incorrect). Daniel had not solved the problem of pressure by this time but again

the work shows that his interest was moving in this direction. His medical work on the flow of blood and blood pressure also gave him an interest in fluid flow. The third part of *Mathematical exercises* was on the Riccati differential equation while the final part was on a geometry question concerning figures bounded by two arcs of a circle.

While in Venice, Daniel had also designed an hour glass to be used at sea so that the trickle of sand was constant even when the ship was rolling in heavy seas. He submitted his work on this to the Paris Academy and in 1725, the year he returned from Italy to Basel, he learnt that he had won the prize of the Paris Academy. Daniel had also attained fame through his work *Mathematical exercises* and on the strength of this he was invited to take up the chair of mathematics at St Petersburg. His brother Nicolaus(II) Bernoulli was also offered a chair of mathematics at St Petersburg so in late 1725 the two brothers travelled to St Petersburg.

Within eight months of their taking up the appointments in St Petersburg Daniel's brother died of fever. Daniel was left, greatly saddened at the loss of his brother and also very unhappy with the harsh climate. He thought of returning to Basel and wrote to his father telling him how unhappy he was in St Petersburg. Johann Bernoulli was able to arrange for one of his best pupils, Leonard Euler, to go to St Petersburg to work with Daniel. Euler arrived in 1727 and this period in St Petersburg, which Daniel left in 1733, was to be his most productive time.

One of the topics which Daniel studied in St Petersburg was that of vibrating systems. As Straub writes in [1]:-

From 1728*, Bernoulli and Euler dominated the mechanics of flexible and elastic bodies, in that year deriving the equilibrium curves for these bodies. ... Bernoulli determined the shape that a perfectly flexible thread assumes when acted upon by forces of which one component is vertical to the curve and the other is parallel to a given direction. Thus, in one stroke he derived the entire series of such curves as the velaria, lintearia, catenaria...*

While in St Petersburg he made one of his most famous discoveries when he defined the simple nodes and the frequencies of oscillation of a system. He showed that the movements of strings of musical instruments are composed of an infinite number of harmonic vibrations all superimposed on the string.

A second important work which Daniel produced while in St Petersburg was one on probability and political economy. Daniel makes the assumption that the moral value of the increase in a person's wealth is inversely proportional to the amount of that wealth. He then assigns probabilities to the various means that a person has to make money and deduces an expectation of increase in moral expectation. Daniel applied some of his deductions to insurance.

Undoubtedly the most important work which Daniel Bernoulli did while in St Petersburg was his work on hydrodynamics. Even the term itself is based on the title of the work which he produced called *Hydrodynamica* and, before he left St Petersburg, Daniel left a draft copy of the book with a printer. However the work was not published until 1738 and although he revised it considerably between 1734 and 1738, it is more the presentation that he changed rather then the substance.

This work contains for the first time the correct analysis of water flowing from a hole in a container. This was based on the principle of conservation of energy which he had studied with his father in 1720. Daniel also discussed pumps and other machines to raise water. One remarkable discovery appears in Chapter 10 of *Hydrodynamica* where Daniel discussed the basis for the kinetic theory of gases. He was able to give the basic laws for the theory of gases and gave, although not in full detail, the equation of state discovered by Van der Waals a century later.

Daniel Bernoulli was not happy in St Petersburg, despite the obvious scientific advantage of working with Euler. By 1731 he was applying for posts in Basel but probability seemed to work against him and he would lose out in the ballot for the post. The post was neither one in mathematics nor physics but Daniel preferred to return to Basel and give lectures on botany rather than remain in St Petersburg. By this time his younger brother

Johann(II) Bernoulli was also with him in St Petersburg and they left St Petersburg in 1733, making visits to Danzig, Hamburg, Holland and Paris before returning to Basel in 1734.

Daniel Bernoulli submitted an entry for the Grand Prize of the Paris Academy for 1734 giving an application of his ideas to astronomy. This had unfortunate consequences since Daniel's father, Johann Bernoulli, also entered for the prize and their entries were declared joint winners of the Grand Prize. The result of this episode of the prize of the Paris Academy had unhappy consequences for Daniel. His father was furious to think that his son had been rated as his equal and this resulted in a breakdown in relationships between the two. The outcome was that Daniel found himself back in Basel but banned from his father's house. Whether this caused Daniel to become less interested in mathematics or whether it was the fact that his academic position was a non mathematical one, certainly Daniel never regained the vigour for mathematical research that he showed in St Petersburg.

Although Daniel had left St Petersburg, he began an immediate correspondence with Euler and the two exchanged many ideas on vibrating systems. Euler used his great analytic skills to put many of Daniel's physical insights into a rigorous mathematical form. Daniel continued to work on polishing his masterpiece *Hydrodynamica* for publication and added a chapter on the force of reaction of a jet of fluid and the force of a jet of water on an inclined plane. In this chapter, Chapter 13, he also discussed applications to the propulsion of ships.

The 1737 prize of the Paris Academy also had a nautical theme, the best shape for a ship's anchor, and Daniel Bernoulli was again the joint winner of this prize, this time jointly with Poleni. *Hydrodynamica* was published in 1738 but, in the following year Johann Bernoulli published *Hydraulica* which is largely based on his son's work but Johann tried to make it look as if Daniel had based *Hydrodynamica* on *Hydraulica* by predating the date of publication on his book to 1732 instead of its real date which is probably 1739. This was a disgraceful attempt by Johann to gain credit for work which was not his and at the same time to discredit his own son and shows the depths to which the bad feeling between them had reached.

It is fair to say that there is no evidence that Daniel was in any way to blame for the breakdown of relationships with his father. Rather the reverse since there is evidence that he tried to mend the relationship with such acts as describing himself on the frontispiece of *Hydrodynamica* as 'Daniel Bernoulli, son of Johann'. Another sign that Daniel was not jealous of members of his own family in the way the Johann Bernoulli and Jacob Bernoulli had been is the fact that he did produce joint work with his younger brother Johann(II) Bernoulli.

Botany lectures were not what Daniel wanted and things became better for him in 1743 when he was able to exchange these for physiology lectures. In 1750, however, he was appointed to the chair of physics and taught physics at Basel for 26 years until 1776. He gave some remarkable physics lectures with experiments performed during the lectures. Based on experimental evidence he was able to conjecture certain laws which were not verified until many years later. Among these was Coulomb's law in electrostatics.

Daniel Bernoulli did produce other excellent scientific work during these many years back in Basel. In total he won the Grand Prize of the Paris Academy 10 times, for topics in astronomy and nautical topics. He won in 1740 (jointly with Euler) for work on Newton's theory of the tides; in 1743 and 1746 for essays on magnetism; in 1747 for a method to determine time at sea; in 1751 for an essay on ocean currents; in 1753 for the effects of forces on ships; and in 1757 for proposals to reduce the pitching and tossing of a ship in high seas.

Another important aspect of Daniel Bernoulli's work that proved important in the development of mathematical physics was his acceptance of many of Newton's theories and his use of these together with the tolls coming from the more powerful calculus of Leibniz. Daniel worked on mechanics and again used the principle of conservation of energy which gave an integral of Newton's basic equations. He also studied the movement of bodies in a resisting medium using Newton's methods.

He also continued to produce good work on the theory of oscillations and in a paper he gave a beautiful account of the oscillation of air in organ pipes. His strengths and weaknesses are summed up by Straub in [1]:-

Bernoulli's active and imaginative mind dealt with the most varied scientific areas. Such wide interests, however, often prevented him from carrying some of his projects to completion. It is especially unfortunate that he could not follow the rapid growth of mathematics that began with the introduction of partial differential equations into mathematical physics. Nevertheless he assured himself a permanent place in the history of science through his work and discoveries in hydrodynamics, his anticipation of the kinetic theory of gases, a novel method for calculating the value of an increase in assets, and the demonstration that the most common movement of a string in a musical instrument is composed of the superposition of an infinite number of harmonic vibrations...

Daniel Bernoulli was much honoured in his own lifetime. He was elected to most of the leading scientific societies of his day including those in Bologna, St Petersburg, Berlin, Paris, London, Bern, Turin, Zurich and Mannheim.

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