Augustin Louis Cauchy

Born: 21 Aug 1789 in Paris, France
Died: 23 May 1857 in Sceaux (near Paris), France

Paris was a difficult place to live in when **Augustin-Louis Cauchy** was a young child due to the political events surrounding the French Revolution. When he was four years old his father, fearing for his life in Paris, moved his family to Arcueil. There things were hard and he wrote in a letter:-

We never have more than a half pound of bread - and sometimes not even that. This we supplement with the little supply of hard crackers and rice that we are allotted.

They soon returned to Paris and Cauchy's father was active in the education of young Augustin-Louis. Laplace and Lagrange were visitors at the Cauchy family home and Lagrange in particular seems to have taken an interest in young Cauchy's mathematical education. Lagrange advised Cauchy's father that his son should obtain a good grounding in languages before starting a serious study of mathematics. In 1802 Augustin-Louis entered the École Centrale du Panthéon where he spent two years studying classical languages.

From 1804 Cauchy attended classes in mathematics and he took the entrance examination for the École Polytechnique in 1805. He was examined by Biot and placed second. At the École Polytechnique he attended courses by Lacroix, de Prony and Hachette while his analysis tutor was Ampère. In 1807 he graduated from the École Polytechnique and entered the engineering school École des Ponts et Chaussées. He was an outstanding student and for his practical work he was assigned to the Ourcq Canal project where he worked under Pierre Girard.

In 1810 Cauchy took up his first job in Cherbourg to work on port facilities for Napoleon's English invasion fleet. He took a copy of Laplace's *Mécanique Céleste* and one of Lagrange's *Théorie des Fonctions* with him. It was a busy time for Cauchy, writing home about his daily duties he said:-

I get up at four o'clock each morning and I am busy from then on. ... I do not get tired of working, on the contrary, it invigorates me and I am in perfect health...

Cauchy was a devout Catholic and his attitude to his religion was already causing problems for him. In a letter written to his mother in 1810 he says:-

So they are claiming that my devotion is causing me to become proud, arrogant and self-infatuated. ... I am now left alone about religion and nobody mentions it to me anymore...

In addition to his heavy workload Cauchy undertook mathematical researches and he proved in 1811 that the angles of a convex polyhedron are determined by its faces. He submitted his first paper on this topic then, encouraged by Legendre and Malus, he submitted a further paper on polygons and polyhedra in 1812. Cauchy felt that he had to return to Paris if he was to make an impression with mathematical research. In September of 1812 he returned to Paris after becoming ill. It appears that the illness was not a physical one and was probably of a psychological nature resulting in severe depression.

Back in Paris Cauchy investigated symmetric functions and submitted a memoir on this topic in November 1812. This was published in the Journal of the École Polytechnique in 1815. However he was supposed to return to Cherbourg in February 1813 when he had recovered his health and this did not fit with his mathematical ambitions. His request to de Prony for an associate professorship at the École des Ponts et

Chaussées was turned down but he was allowed to continue as an engineer on the Ourcq Canal project rather than return to Cherbourg. Pierre Girard was clearly pleased with his previous work on this project and supported the move.

An academic career was what Cauchy wanted and he applied for a post in the Bureau des Longitudes. He failed to obtain this post, Legendre being appointed. He also failed to be appointed to the geometry section of the Institute, the position going to Poinsot. Cauchy obtained further sick leave, having unpaid leave for nine months, then political events prevented work on the Ourcq Canal so Cauchy was able to devote himself entirely to research for a couple of years.

Other posts became vacant but one in 1814 went to Ampère and a mechanics vacancy at the Institute, which had occurred when Napoleon Bonaparte resigned, went to Molard. In this last election Cauchy did not receive a single one of the 53 votes cast. His mathematical output remained strong and in 1814 he published the memoir on definite integrals that later became the basis of his theory of complex functions.

In 1815 Cauchy lost out to Binet for a mechanics chair at the École Polytechnique, but then was appointed assistant professor of analysis there. He was responsible for the second year course. In 1816 he won the Grand Prix of the French Academy of Sciences for a work on waves. He achieved real fame however when he submitted a paper to the Institute solving one of Fermat's claims on polygonal numbers made to Mersenne. Politics now helped Cauchy into the Academy of Sciences when Carnot and Monge fell from political favour and were dismissed and Cauchy filled one of the two places.

In 1817 when Biot left Paris for an expedition to the Shetland Islands in Scotland Cauchy filled his post at the Collège de France. There he lectured on methods of integration which he had discovered, but not published, earlier. Cauchy was the first to make a rigorous study of the conditions for convergence of infinite series in addition to his rigorous definition of an integral. His text *Cours d'analyse* in 1821 was designed for students at École Polytechnique and was concerned with developing the basic theorems of the calculus as rigorously as possible. He began a study of the calculus of residues in 1826 in *Sur un nouveau genre de calcul analogue au calcul infinitésimal* while in 1829 in *Leçons sur le Calcul Différentiel* he defined for the first time a complex function of a complex variable.

Cauchy did not have particularly good relations with other scientists. His staunchly Catholic views had him involved on the side of the Jesuits against the Académie des Sciences. He would bring religion into his scientific work as for example he did on giving a report on the theory of light in 1824 when he attacked the author for his view that Newton had not believed that people had souls. He was described by a journalist who said:-

... it is certain a curious thing to see an academician who seemed to fulfil the respectable functions of a missionary preaching to the heathens.

An example of how Cauchy treated colleagues is given by Poncelet whose work on projective geometry had, in 1820, been criticised by Cauchy:-

... I managed to approach my too rigid judge at his residence ... just as he was leaving ... During this very short and very rapid walk, I quickly perceived that I had in no way earned his regards or his respect as a scientist ... without allowing me to say anything else, he abruptly walked off, referring me to the forthcoming publication of his Leçons à 'École Polytechnique where, according to him, 'the question would be very properly explored'.

Again his treatment of Galois and Abel during this period was unfortunate. Abel, who visited the Institute in 1826, wrote of him:-

Cauchy is mad and there is nothing that can be done about him, although, right now, he is the only one who knows how mathematics should be done.

Belhoste in [4] says:-

When Abel's untimely death occurred on April 6, 1829*, Cauchy still had not given a report on the* 1826 *paper, in spite of several protests from Legendre. The report he finally did give, on June* 29, 1829*, was hasty, nasty, and superficial, unworthy of both his own brilliance and the real importance of the study he had judged.*

By 1830 the political events in Paris and the years of hard work had taken their toll and Cauchy decided to take a break. He left Paris in September 1830, after the revolution of July, and spent a short time in Switzerland. There he was an enthusiastic helper in setting up the Académie Helvétique but this project collapsed as it became caught up in political events.

Political events in France meant that Cauchy was now required to swear an oath of allegiance to the new regime and when he failed to return to Paris to do so he lost all his positions there. In 1831 Cauchy went to Turin and after some time there he accepted an offer from the King of Piedmont of a chair of theoretical physics. He taught in Turin from 1832. Menabrea attended these courses in Turin and wrote that the courses:-

were very confused, skipping suddenly from one idea to another, from one formula to the next, with no attempt to give a connection between them. His presentations were obscure clouds, illuminated from time to time by flashes of pure genius. ... of the thirty who enrolled with me, I was the only one to see it through.

In 1833 Cauchy went from Turin to Prague in order to follow Charles X and to tutor his grandson. However he was not very successful in teaching the prince as this description shows:-

... exams .. were given each Saturday. ... When questioned by Cauchy on a problem in descriptive geometry, the prince was confused and hesitant. ... There was also material on physics and chemistry. As with mathematics, the prince showed very little interest in these subjects. Cauchy became annoyed and screamed and yelled. The queen sometimes said to him, soothingly, smilingly, 'too loud, not so loud'.

While in Prague Cauchy had one meeting with Bolzano, at Bolzano's request, in 1834. In [16] and [18] there are discussions on how much Cauchy's definition of continuity is due to Bolzano, Freudenthal's view in [18] that Cauchy's definition was formed before Bolzano's seems the more convincing.

Cauchy returned to Paris in 1838 and regained his position at the Academy but not his teaching positions because he had refused to take an oath of allegiance. De Prony died in 1839 and his position at the Bureau des Longitudes became vacant. Cauchy was strongly supported by Biot and Arago but Poisson strongly opposed him. Cauchy was elected but, after refusing to swear the oath, was not appointed and could not attend meetings or receive a salary.

In 1843 Lacroix died and Cauchy became a candidate for his mathematics chair at the Collège de France. Liouville and Libri were also candidates. Cauchy should have easily been appointed on his mathematical abilities but his political and religious activities, such as support for the Jesuits, became crucial factors. Libri was chosen, clearly by far the weakest of the three mathematically, and Liouville wrote the following day that he was:-

deeply humiliated as a man and as a mathematician by what took place yesterday at the Collège de France.

During this period Cauchy's mathematical output was less than in the period before his self-imposed exile. He did important work on differential equations and applications to mathematical physics. He also wrote on mathematical astronomy, mainly because of his candidacy for positions at the Bureau des Longitudes. The 4-volume text *Exercises d'analyse et de physique mathématique* published between 1840 and 1847 proved extremely important.

When Louis Philippe was overthrown in 1848 Cauchy regained his university positions. However he did not change his views and continued to give his colleagues problems. Libri, who had been appointed in the political way described above, resigned his chair and fled from France. Partly this must have been because he was about to be prosecuted for stealing valuable books. Liouville and Cauchy were candidates for the chair again in 1850 as they had been in 1843. After a close run election Liouville was appointed. Subsequent attempts to reverse this decision led to very bad relations between Liouville and Cauchy.

Another, rather silly, dispute this time with Duhamel clouded the last few years of Cauchy's life. This dispute was over a priority claim regarding a result on inelastic shocks. Duhamel argued with Cauchy's claim to have been the first to give the results in 1832. Poncelet referred to his own work of 1826 on the subject and Cauchy was shown to be wrong. However Cauchy was never one to admit he was wrong. Valson writes in [7]:-

...the dispute gave the final days of his life a basic sadness and bitterness that only his friends were aware of...

Also in [7] a letter by Cauchy's daughter describing his death is given:-

Having remained fully alert, in complete control of his mental powers, until 3.30 *a.m.. my father suddenly uttered the blessed names of Jesus, Mary and Joseph. For the first time, he seemed to be aware of the gravity of his condition. At about four o'clock, his soul went to God. He met his death with such calm that made us ashamed of our unhappiness.*

Numerous terms in mathematics bear Cauchy's name:- the Cauchy integral theorem, in the theory of complex functions, the Cauchy-Kovalevskaya existence theorem for the solution of partial differential equations, the Cauchy-Riemann equations and Cauchy sequences. He produced 789 mathematics papers, an incredible achievement. This achievement is summed up in [4] as follows:-

... such an enormous scientific creativity is nothing less than staggering, for it presents research on all the then-known areas of mathematics ... in spite of its vastness and rich multifaceted character, Cauchy's scientific works possess a definite unifying theme, a secret wholeness. ... Cauchy's creative genius found broad expression not only in his work on the foundations of real and complex analysis, areas to which his name is inextricably linked, but also in many other fields. Specifically, in this connection, we should mention his major contributions to the development of mathematical physics and to theoretical mechanics... we mention ... his two theories of elasticity and his investigations on the theory of light, research which required that he develop whole new mathematical techniques such as Fourier transforms, diagonalisation of matrices, and the calculus of residues.

His collected works, *Oeuvres complètes d'Augustin Cauchy* (1882-1970), were published in 27 volumes.

Article by: *J J O'Connor* and *E F Robertson*

January 1997