Johann Peter Gustav Lejeune Dirichlet

Born: 13 Feb 1805 in Düren, French Empire (now Germany)
Died: 5 May 1859 in Göttingen, Hanover (now Germany)

Lejeune Dirichlet's family came from the Belgium town of Richelet where Dirichlet's grandfather lived. This explains the origin of his name which comes from "Le jeune de Richelet" meaning "Young from Richelet". Many details of the Dirichlet family are given in [6] where it is shown that the Dirichlets came from the neighbourhood of Liège in Belgium and not, as many had claimed, from France.

His father was the postmaster of Düren, the town of his birth situated about halfway between Aachen and Cologne. Even before he entered the Gymnasium in Bonn in 1817, at the age of 12, he had developed a passion for mathematics and spent his pocket-money on buying mathematics books. At the Gymnasium he was a model pupil being [1]:-

... an unusually attentive and well-behaved pupil who was particularly interested in history as well as mathematics.

After two years at the Gymnasium in Bonn his parents decided that they would rather have him attend the Jesuit College in Cologne and there he had the good fortune to be taught by Ohm. By the age of 16 Dirichlet had completed his school qualifications and was ready to enter university. However, the standards in German universities were not high at this time so Dirichlet decided to study in Paris. It is interesting to note that some years later the standards in German universities would become the best in the world and Dirichlet himself would play a hand in the transformation.

Dirichlet set off for France carrying with him Gauss's *Disquisitiones arithmeticae* a work he treasured and kept constantly with him as others might do with the Bible. In Paris by May 1822, Dirichlet soon contracted smallpox. It did not keep him away from his lectures in the Collège de France and the Faculté des Sciences for long and soon he could return to lectures. He had some of the leading mathematicians as teachers and he was able to profit greatly from the experience of coming in contact with Biot, Fourier, Francoeur, Hachette, Laplace, Lacroix, Legendre, and Poisson.

From the summer of 1823 Dirichlet was employed by General Maximilien Sébastien Foy, living in his house in Paris. General Foy had been a major figure in the army during the Napoleonic Wars, retiring after Napoleon's defeat at Waterloo. In 1819 he was elected to the Chamber of Deputies where he was leader of the liberal opposition until his death. Dirichlet was very well treated by General Foy, he was well paid yet treated like a member of the family. In return Dirichlet taught German to General Foy's wife and children.

Dirichlet's first paper was to bring him instant fame since it concerned the famous Fermat's Last Theorem. The theorem claimed that for $n > 2$ there are no non-zero integers x, y, z such that $x^n + y^n = z^n$. The cases $n = 3$ and $n = 4$ had been proved by Euler and Fermat, and Dirichlet attacked the theorem for $n = 5$. Now if $n = 5$ then one of x, y, z is even and one is divisible by 5. There are two cases: case 1 is when the number divisible by 5 is even, while case 2 is when the even number and the one divisible by 5 are distinct. Dirichlet proved case 1 and presented his paper to the Paris Academy in July 1825. Legendre was appointed one of the referees and he was able to prove case 2 thus completing the proof for $n = 5$. The complete proof was published in September 1825. In fact Dirichlet was able to complete his own proof of the $n = 5$ case with an argument for case 2 which was an extension of his own argument for case 1. It is worth noting that Dirichlet made a later contribution proving the $n = 14$ case (a near miss for the $n = 7$ case!).

On 28 November 1825 General Foy died and Dirichlet decided to return to Germany. He was encouraged in this by Alexander von Humboldt who made recommendations on his behalf. There was a problem for Dirichlet since in order to teach in a German university he needed an habilitation. Although Dirichlet could easily submit an habilitation thesis, this was not allowed since he did not hold a doctorate, nor could he speak Latin, a requirement in the early nineteenth century. The problem was nicely solved by the University of Cologne giving Dirichlet an honorary doctorate, thus allowing him to submit his habilitation thesis on polynomials with a special class of prime divisors to the University of Breslau. There was, however, much controversy over Dirichlet's appointment and the large correspondence between German professors both for and against his appointment is considered in [15].

From 1827 Dirichlet taught at Breslau but Dirichlet encountered the same problem which made him choose Paris for his own education, namely that the standards at the university were low. Again with von Humboldt's help, he moved to the Berlin in 1828 where he was appointed at the Military College. The Military College was not the attraction, of course, rather it was that Dirichlet had an agreement that he would be able to teach at the University of Berlin. Soon after this he was appointed a professor at the University of Berlin where he remained from 1828 to 1855. He retained his position in the Military College which made his teaching and other administrative duties rather heavier than he would have liked.

Dirichlet was appointed to the Berlin Academy in 1831 and an improving salary from the university put him in a position to marry, and he married Rebecca Mendelssohn, one of the composer Felix Mendelssohn's two sisters. Dirichlet had a lifelong friend in Jacobi, who taught at Königsberg, and the two exerted considerable influence on each other in their researches in number theory.

In the 1843 Jacobi became unwell and diabetes was diagnosed. He was advised by his doctor to spend time in Italy where the climate would help him recover. However, Jacobi was not a wealthy man and Dirichlet, after visiting Jacobi and discovering his plight, wrote to Alexander von Humboldt asking him to help obtain some financial assistance for Jacobi from Friedrich Wilhelm IV. Dirichlet then made a request for assistance from Friedrich Wilhelm IV, supported strongly by Alexander von Humboldt, which was successful. Dirichlet obtained leave of absence from Berlin for eighteen months and in the autumn of 1843 set off for Italy with Jacobi and Borchardt. After stopping in several towns and attending a mathematical meeting in Lucca, they arrived in Rome on 16 November 1843. Schläfli and Steiner were also with them, Schläfli's main task being to act as their interpreter but he studied mathematics with Dirichlet as his tutor.

Dirichlet did not remain in Rome for the whole period, but visited Sicily and then spent the winter of 1844/45 in Florence before returning to Berlin in the spring of 1845. Dirichlet had a high teaching load at the University of Berlin, being also required to teach in the Military College and in 1853 he complained in a letter to his pupil Kronecker that he had thirteen lectures a week to give in addition to many other duties. It was therefore something of a relief when, on Gauss's death in 1855, he was offered his chair at Göttingen.

Dirichlet did not accept the offer from Göttingen immediately but used it to try to obtain better conditions in Berlin. He requested of the Prussian Ministry of Culture that he be allowed to end lecturing at the Military College. However he received no quick reply to his modest request so he wrote to Göttingen accepting the offer of Gauss's chair. After he had accepted the Göttingen offer the Prussian Ministry of Culture did try to offer him improved conditions and salary but this came too late.

The quieter life in Göttingen seemed to suit Dirichlet. He had more time for research and some outstanding research students. However, sadly he was not to enjoy the new life for long. In the summer of 1858 he lectured at a conference in Montreux but while in the Swiss town he suffered a heart attack. He returned to Göttingen, with the greatest difficulty, and while gravely ill had the added sadness that his wife died of a stroke.

We should now look at Dirichlet's remarkable contributions to mathematics. We have already commented on his contributions to Fermat's Last Theorem made in 1825. Around this time he also published a paper inspired by Gauss's work on the law of biquadratic reciprocity. Details are given in [13] where Rowe discusses the importance of the intellectual and personal relationship between Gauss and Dirichlet.

He proved in 1837 that in any arithmetic progression with first term coprime to the difference there are infinitely many primes. This had been conjectured by Gauss. Davenport wrote in 1980 (see [16]):-

Analytic number theory may be said to begin with the work of Dirichlet, and in particular with Dirichlet's memoir of 1837 *on the existence of primes in a given arithmetic progression.*

Shortly after publishing this paper Dirichlet published two further papers on analytic number theory, one in 1838 with the next in the following year. These papers introduce Dirichlet series and determine, among other things, the formula for the class number for quadratic forms.

His work on units in algebraic number theory *Vorlesungen über Zahlentheorie* (published 1863) contains important work on ideals. He also proposed in 1837 the modern definition of a function:-

If a variable y is so related to a variable x that whenever a numerical value is assigned to x, there is a rule according to which a unique value of y is determined, then y is said to be a function of the independent variable x.

In mechanics he investigated the equilibrium of systems and potential theory. These investigations began in 1839 with papers which gave methods to evaluate multiple integrals and he applied this to the problem of the gravitational attraction of an ellipsoid on points both inside and outside. He turned to Laplace's problem of proving the stability of the solar system and produced an analysis which avoided the problem of using series expansion with quadratic and higher terms disregarded. This work led him to the Dirichlet problem concerning harmonic functions with given boundary conditions. Some work on mechanics later in his career is of quite outstanding importance. In 1852 he studied the problem of a sphere placed in an incompressible fluid, in the course of this investigation becoming the first person to integrate the hydrodynamic equations exactly.

Dirichlet is also well known for his papers on conditions for the convergence of trigonometric series and the use of the series to represent arbitrary functions. These series had been used previously by Fourier in solving differential equations. Dirichlet's work is published in *Crelle's Journal* in 1828. Earlier work by Poisson on the convergence of Fourier series was shown to be non-rigorous by Cauchy. Cauchy's work itself was shown to be in error by Dirichlet who wrote of Cauchy's paper:-

The author of this work himself admits that his proof is defective for certain functions for which the convergence is, however, incontestable.

Because of this work Dirichlet is considered the founder of the theory of Fourier series. Riemann, who was a student of Dirichlet, wrote in the introduction to his habilitation thesis on Fourier series that it was Dirichlet [11]:-

... who wrote the first profound paper about the subject.

In [1] Dirichlet's character and teaching qualities are summed up as follows:-

He was an excellent teacher, always expressing himself with great clarity. His manner was modest; in his later years he was shy and at times reserved. He seldom spoke at meetings and was reluctant to make public appearances.

At age 45 Dirichlet was described by Thomas Hirst as follows:-

He is a rather tall, lanky-looking man, with moustache and beard about to turn grey with a somewhat harsh voice and rather deaf. He was unwashed, with his cup of coffee and cigar. One of his failings is forgetting time, he pulls his watch out, finds it past three, and runs out without even finishing the sentence.

Koch, in [11], sums up Dirichlet's contribution writing that:-

... important parts of mathematics were influenced by Dirichlet. His proofs characteristically started with surprisingly simple observations, followed by extremely sharp analysis of the remaining problem. With Dirichlet began the golden age of mathematics in Berlin.

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