Adrien-Marie Legendre

Born: 18 Sept 1752 in Paris, France
Died: 10 Jan 1833 in Paris, France

Adrien-Marie Legendre would perhaps have disliked the fact that this article contains details of his life for Poisson wrote of him in [12]:-

Our colleague has often expressed the desire that, in speaking of him, it would only be the matter of his works, which are, in fact, his entire life.

It is not surprising that, given these views of Legendre, there are few details of his early life. We have given his place of birth as Paris, as given in [1] and [2], but there is some evidence to suggest that he was born in Toulouse and the family moved to Paris when he was very young. He certainly came from a wealthy family and he was given a top quality education in mathematics and physics at the Collège Mazarin in Paris.

In 1770, at the age of 18, Legendre defended his thesis in mathematics and physics at the Collège Mazarin but this was not quite as grand an achievement as it sounds to us today, for this consisted more of a plan of research rather than a completed thesis. In the thesis he listed the literature that he would study and the results that he would be aiming to prove. With no need for employment to support himself, Legendre lived in Paris and concentrated on research.

From 1775 to 1780 he taught with Laplace at École Militaire where his appointment was made on the advice of d'Alembert. He then decided to enter for the 1782 prize on projectiles offered by the Berlin Academy. The actual task was stated as follows:-

Determine the curve described by cannonballs and bombs, taking into consideration the resistance of the air; give rules for obtaining the ranges corresponding to different initial velocities and to different angles of projection.

His essay *Recherches sur la trajectoire des projectiles dans les milieux résistants* won the prize and launched Legendre on his research career. In 1782 Lagrange was Director of Mathematics at the Academy in Berlin and this brought Legendre to his attention. He wrote to Laplace asking for more information about the prize winning young mathematician.

Legendre next studied the attraction of ellipsoids. He gave a proof of a result due to Maclaurin, that the attractions at an external point lying on the principal axis of two confocal ellipsoids was proportional to their masses. He then introduced what we call today the Legendre functions and used these to determine, using power series, the attraction of an ellipsoid at any exterior point. Legendre submitted his results to the Académie des Sciences in Paris in January 1783 and these were highly praised by Laplace in his report delivered to the Académie in March. Within a few days, on 30 March, Legendre was appointed an adjoint in the Académie des Sciences filling the place which had become vacant when Laplace was promoted from adjoint to associé earlier that year.

Over the next few years Legendre published work in a number of areas. In particular he published on celestial mechanics with papers such as *Recherches sur la figure des planètes* in 1784 which contains the Legendre polynomials; number theory with, for example, *Recherches d'analyse indéterminée* in 1785; and the theory of elliptic functions with papers on integrations by elliptic arcs in 1786.

The 1785 paper on number theory contains a number of important results such as the law of quadratic reciprocity for residues and the results that every arithmetic series with the first term coprime to the common difference contains an infinite number of primes. Of course today we attribute the law of quadratic reciprocity to Gauss and the theorem concerning primes in an arithmetic progression to Dirichlet. This is fair since Legendre's proof of quadratic reciprocity was unsatisfactory, while he offered no proof of the theorem on primes in an arithmetic progression. However, these two results are of great importance and credit should go to Legendre for his work on them, although he was not the first to state the law of quadratic reciprocity since it occurs in Euler's work of 1751 and also of 1783 (see [15]).

Legendre's career in the Académie des Sciences progressed in a satisfactory manner. He became an associé in 1785 and then in 1787 he was a member of the team whose task it was to work with the Royal Observatory at Greenwich in London on measurements of the Earth involving a triangulation survey between the Paris and Greenwich observatories. This work resulted in his election to the Royal Society of London in 1787 and also to an important publication *Mémoire sur les opérations trigonométriques dont les résultats dépendent de la figure de la terre* which contains Legendre's theorem on spherical triangles.

On 13 May 1791 Legendre became a member of the committee of the Académie des Sciences with the task to standardise weights and measures. The committee worked on the metric system and undertook the necessary astronomical observations and triangulations necessary to compute the length of the metre. At this time Legendre was also working on his major text *Eléments de géométrie* which he had been encouraged to write by Condorcet. However the Académie des Sciences was closed due to the Revolution in 1793 and Legendre had special difficulties since he lost the capital which provided him with a comfortable income. He later wrote to Jacobi explaining his personal circumstances around this time (see [1]):-

I married following a bloody revolution that had destroyed my small fortune; we had great problems and some very difficult moments, but my wife staunchly helped me to put my affairs in order little by little and gave me the tranquillity necessary for my customary work and for writing new works which have steadily increased my reputation.

Following the work of the committee on the decimal system on which Legendre had served, de Prony in 1792 began a major task of producing logarithmic and trigonometric tables, the Cadastre. Legendre and de Prony headed the mathematical section of this project along with Carnot and other mathematicians. They had between 70 to 80 assistants and the work was undertaken over a period of years, being completed in 1801.

In 1794 Legendre published *Eléments de géométrie* which was the leading elementary text on the topic for around 100 years. The work is described in [2]:-

In his "Eléments" Legendre greatly rearranged and simplified many of the propositions from Euclid's "Elements" to create a more effective textbook. Legendre's work replaced Euclid's "Elements" as a textbook in most of Europe and, in succeeding translations, in the United States and became the prototype of later geometry texts. In "Eléments" Legendre gave a simple proof that π *is irrational, as well as the first proof that* π^2 *is irrational, and conjectured that* π *is not the root of any algebraic equation of finite degree with rational coefficients.*

In 1795 the Académie des Sciences was reopened as the Institut National des Sciences et des Arts and from then until 1806 it met in the Louvre. Each section of the Institut contained six places, and Legendre was one of the six in the mathematics section. In 1803 Napoleon reorganised the Institut and a geometry section was created and Legendre was put into this section.

Legendre published a book on determining the orbits of comets in 1806. In this he wrote:-

I have thought that what there was better to do in the problem of comets was to start out from the immediate data of observation, and to use all means to simplify as much as possible the formulas and the equations which serve to determine the elements of the orbit.

His method involved three observations taken at equal intervals and he assumed that the comet followed a parabolic path so that he ended up with more equations than there were unknowns. He applied his methods to the data known for two comets. In an appendix Legendre gave the least squares method of fitting a curve to the data available. However, Gauss published his version of the least squares method in 1809 and, while acknowledging that it appeared in Legendre's book, Gauss still claimed priority for himself. This greatly hurt Legendre who fought for many years to have his priority recognised.

In 1808 Legendre published a second edition of his *Théorie des nombres* which was a considerable improvement on the first edition of 1798. For example Gauss had proved the law of quadratic reciprocity in 1801 after making critical remarks about Legendre's proof of 1785 and Legendre's much improved proof of 1798 in the first edition of *Théorie des nombres.* Gauss was correct, but one could understand how hurtful Legendre must have found an attack on the rigour of his results by such a young man. Of course Gauss did not state that he was improving Legendre's result but rather claimed the result for himself since his was the first completely rigorous proof. Legendre later wrote (see [20]):-

This excessive impudence is unbelievable in a man who has sufficient personal merit not to have need of appropriating the discoveries of others.

To his credit Legendre used Gauss's proof of quadratic reciprocity in the 1808 edition of *Théorie des nombres* giving proper credit to Gauss. The 1808 edition of *Théorie des nombres* also contained Legendre's estimate for $p(n)$ the number of primes $\le n$ of $p(n) = n/(\log(n) - 1.08366)$. Again Gauss would claim that he had obtained the law for the asymptotic distribution of primes before Legendre, but certainly it was Legendre who first brought these ideas to the attention of mathematicians.

Legendre's major work on elliptic functions in *Exercices du Calcul Intégral* appeared in three volumes in 1811, 1817, and 1819. In the first volume Legendre introduced basic properties of elliptic integrals and also of beta and gamma functions. More results on beta and gamma functions appeared in the second volume together with applications of his results to mechanics, the rotation of the Earth, the attraction of ellipsoids and other problems. The third volume was largely devoted to tables of elliptic integrals.

In November 1824 he decided to reprint a new edition but he was not happy with this work by September 1825 publication began of his new work *Traité des Fonctions Elliptiques* again in three volumes of 1825, 1826, and 1830. This new work covered similar material to the original but the material was completely reorganised. However, despite spending 40 years working on elliptic functions, Legendre never gained the insight of Jacobi and Abel and the independent work of these two mathematicians was making Legendre's new three volume work obsolete almost as soon as it was published.

Legendre's attempt to prove the parallel postulate extended over 30 years. However as stated in [1] his attempts:-

... all failed because he always relied, in the last analysis, on propositions that were "evident" from the Euclidean point of view.

In 1832 (the year Bolyai published his work on non-euclidean geometry) Legendre confirmed his absolute belief in Euclidean space when he wrote:-

It is nevertheless certain that the theorem on the sum of the three angles of the triangle should be considered one of those fundamental truths that are impossible to contest and that are an enduring example of mathematical certitude.

In 1824 Legendre refused to vote for the government's candidate for the Institut National. Abel wrote in October 1826:-

Legendre is an extremely amiable man, but unfortunately as old as the stones.

As a result of Legendre's refusal to vote for the government's candidate in 1824 his pension was stopped and he died in poverty.

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