# John Napier

**Born: 1550 in Merchiston Castle, Edinburgh, Scotland**
**Died: 4 April 1617 in Edinburgh, Scotland**

**John Napier**'s father, Archibald Napier, was an important man in late 16<sup>th</sup> century Scotland. His family had owned the Merchiston estate from the 1430s when one of his ancestors acquired the estate, becoming the first Napare of Merchiston. (We shall comment shortly on the different spellings of Napier's name.) The family also owned estates at Lennox and at Menteith and a residence at Gartness. Archibald Napier married Janet Bothwell, the sister of the Bishop of Orkney, in 1549 when he was only 15 years old. Their son John Napier was born the following year. Archibald Napier was a justice-depute and was knighted in 1565. He was appointed Master of the Mint in 1582.

Before continuing we should comment on the spelling of John Napier. The name John is most easily dealt with as John Napier, and almost everyone else around his time, used the old spelling "Jhone". His surname appears in a large variety of different spellings. The forms Napeir, Nepair, Nepeir, Neper, Napare, Naper, Naipper are all seen but John Napier would most commonly have been written Jhone Neper at that time. The only form of Napier that we are sure would not have been used in Napier's lifetime was the present modern spelling "Napier"!

Little is known about John Napier's early years. One of the few scraps of information that we have is from a letter from the Bishop of Orkney, John's uncle, to Archibald Napier written when John was eleven years old:-

*I pray you, sir, to send your son John to school; over to France or Flanders; for he cannot learn well at home nor get profit in this most perilous world - that he may be saved in it; - that he may seek honour and profit as I do not doubt that he will...*

This is a translation of the old Scots that the Bishop of Orkney actually wrote. For those interested the original version reads:-

*I pray you, schir, to send your son Jhone to the schuyllis; oyer to France or Flandaris; for he can leyr na guid at hame, nor get na proffeitt in this maist perullous worlde ...*

Napier was educated at St Andrews University, entering the university in 1563 at the age of 13. His mother arranged for him to live in St Salvator's College and special arrangements were made for the Principal of the University, John Rutherford, to take care of him personally. Napier's name appears on the matriculation roll of St Salvator's College for 1563. Shortly after Napier matriculated his mother died. We know that Napier spent some time at St Andrews University and he wrote himself many years later that it was in St Andrews that he first became passionately interested in theology.

However Napier's name does not appear in the list of those being awarded degrees in the subsequent years so he must have left St Andrews to study in Europe before completing a degree. Of other facts we can also be certain. Napier did not acquire his knowledge of higher mathematics at St Andrews nor did he acquire his deep knowledge of classical literature there. Both these must have been acquired during his studies in Europe but no record exists to show where he studied, although the University of Paris is highly likely and it is also probable that he spent some time in Italy and the Netherlands.

By 1571 Napier had returned to Scotland for he was present at his father's second marriage which took place in that year. It was in 1571 that Napier himself began to make arrangements for his own marriage but it was at

nearly two years before that took place. In 1572 most of the estates of the Napier family were made over to John Napier and a castle was planned for the estate at Gartness.

When the castle was completed in 1574, Napier and his wife took up residence there. Napier devoted himself to running his estates. This task he took very seriously and, being a great genius as an inventor, he applied his skills to these tasks. He approached agriculture in a scientific way and he experimented with:-

*... improving and manuring of all sorts of field land with common salts, whereby the same may bring forth in more abundance, both of grass and corn of all sorts, and far cheaper than by the common way of dunging used heretofore in Scotland.*

The above is quoted in [12] without reference to its origin.

Napier took part in the religious controversies of the time. He was a fervent Protestant and published, what he considered his most important work, the *Plaine Discovery of the Whole Revelation of St. John* (1593).

Napier had been a fanatical Protestant from his days as an undergraduate at St Andrews. He wrote the *Plaine Discovery of the Whole Revelation of St. John* according to his preface:-

*... for preventing the apparent danger of Papistry arising within this Island...*

In fact there were good reasons why Napier thought that a change in the religious situation in Scotland might occur, for there had, for some time, been rumours that Philip of Spain might invade Scotland. The *Plaine Discovery of the Whole Revelation of St. John* did gain Napier quite a reputation, not only within Scotland, but also on the Continent after the work was translated into Dutch, French and German. Gibson, in [12], remarks however:-

*... I suppose that there are few indeed of the present generation who have read, or even heard of, the book; whatever its merits may have been they do not appeal to the modern mind...*

Napier's study of mathematics was only a hobby and in his mathematical works he writes that he often found it hard to find the time for the necessary calculations between working on theology. He is best known, however, for his invention of logarithms but his other mathematical contributions include a mnemonic for formulas used in solving spherical triangles, two formulas known as Napier's analogies used in solving spherical triangles and an invention called Napier's bones used for mechanically multiplying dividing and taking square roots and cube roots. Napier also found exponential expressions for trigonometric functions, and introduced the decimal notation for fractions.

Much of Napier's work on logarithms seems to have been done while he was living at Gartness. The *Statistical Account* (Vol. xvi, page 108) contains the following:-

*Adjoining the mill at Gartness are the remains of an old house in which John Napier of Merchiston, Inventor of Logarithms, resided a great part of his time (some years) when he was making his calculations. It is reported that the noise of the cascade, being constant, never gave him uneasiness, but that the clack of the mill, which was only occasional, greatly disturbed his thoughts. He was therefore, when in deep study, sometimes under the necessity of desiring the miller to stop the mill that the train of his ideas might not be interrupted.*

Napier's discussion of logarithms appears in *Mirifici logarithmorum canonis descriptio* in 1614. Two years later an English translation of Napier's original Latin text was published, translated by Edward Wright. In the preface of the book Napier explains his thinking behind his great discovery (we quote from the English translation of 1616 of the original Latin of 1614):-

*Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances. And having thought upon many things to this purpose, I found at length some excellent brief rules to be treated of (perhaps) hereafter. But amongst all, none more profitable than this which together with the hard and tedious multiplications, divisions, and extractions of roots, doth also cast away from the work itself even the very numbers themselves that are to be multiplied, divided and resolved into roots, and putteth other numbers in their place which perform as much as they can do, only by addition and subtraction, division by two or division by three.*

Unlike the logarithms used today, Napier's logarithms are not really to any base although in our present terminology it is not unreasonable (but perhaps a little misleading) to say that they are to base $1/e$. Certainly they involve a constant $10^7$ which arose from the construction in a way that we will now explain. Napier did not think of logarithms in an algebraic way, in fact algebra was not well enough developed in Napier's time to make this a realistic approach. Rather he thought by dynamical analogy. Consider two lines $AB$ of fixed length and $A'X$ of infinite length. Points $C$ and $C'$ begin moving simultaneously to the right, starting at $A$ and $A'$ respectively with the same initial velocity; $C'$ moves with uniform velocity and $C$ with a velocity which is equal to the distance $CB$. Napier defined $A'C'$ $(= y)$ as the logarithm of $BC$ $(= x)$, that is

$y = \text{Nap.log } x$.

Napier chose the length $AB$ to be $10^7$, based on the fact that the best tables of sines available to him were given to seven decimal places and he thought of the argument $x$ as being of the form $10^2.\sin X$.

The fact that Nap.log 1 does not equal 0 is a major difficulty which make Nap.logs much less convenient for calculations than our logs. A change to logs with log 1 = 0 came about in discussions between Napier and Briggs. Briggs read Napier's 1614 Latin text and, on the 10 March 1615 wrote in a letter to a friend:-

*Napper, lord of Markinston, hath set my head and hands a work with his new and admirable logarithms. I hope to see him this summer, if it please God, for I never saw a book which pleased me better or made me more wonder.*

In fact Briggs did make the difficult journey from London to Edinburgh to see Napier in the summer of 1615 (would he have dreamed that now it takes 4 hours by train, rather than at least 4 days by horse and coach in those times). A description of their meeting was told by John Marr to William Lilly who writes the following (see [12]):-

*Mr Briggs appoints a certain day when to meet at Edinburgh; but failing thereof, Merchiston was fearful he would not come. It happened one day as John Marr and the Lord Napier were speaking of Mr Briggs, "Oh! John," saith Merchiston, "Mr Briggs will not come now"; at the very instant one knocks at the gate, John Marr hastened down and it proved to be Mr Briggs to his great contentment. He brings Mr Briggs into my Lord's chamber, where almost one quarter of an hour was spent, each beholding other with admiration, before one word was spoken. At last Mr Briggs began, -"My Lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help unto astronomy, viz. the Logarithms ...*

Briggs had suggested to Napier in a letter sent before their meeting that logs should be (in our terminology) to base 10 and Briggs had begun to construct tables. Napier replied that he had the same idea but ([12]):-

*... he could not, on account of ill-health and for other weighty reasons undertake the construction of new tables.*

At their meeting Napier suggested to Briggs the new tables should be constructed with base 10 and with log 1 = 0, and indeed Briggs did construct such tables. In fact Briggs spent a month with Napier on his first visit of 1615, made a second journey from London to Edinburgh to visit Napier again in 1616 and would have made yet a third visit the following year but Napier died in the spring before the planned summer visit.

Napier presented a mechanical means of simplifying calculations in his *Rabdologiae* published in 1617. He described a method of multiplication using "numbering rods" with numbers marked off on them. The reason for publishing the work is given by Napier in the dedication, where he says that so many of his friends, to whom he had shown the numbering rods, were so pleased with them that they were already becoming widely used, even beginning to be used in foreign countries.

Napier's numbering rods were made of ivory, so that they looked like bones which explains why they are now known as Napier's bones. To multiply numbers the bones were placed side by side and the appropriate products read off. Glaisher described how to use Napier's bones in an article he wrote for *Encyclopaedia Britannica* and this description is quoted in [10]. Napier's bones are also described in [6], [16] and [19].

It would be surprising if a man of such great an intellect as Napier did not appear rather strange to his contemporaries and, given the superstitious age in which he lived, strange stories began to circulate. Many traditions suggest that Napier was

*... in league with the powers of darkness...*

and these are taken seriously in the biased biography [7] written by Mark Napier, one of John Napier's descendants. Mark Napier suggests that John Napier deliberately played upon the primitive beliefs of his servants by going round with a cock which he had covered in soot. Even the *Statistical Account* (quoted above) says:-

[*Napier*] *used frequently to walk out in his nightgown and cap. This, with some things which to the vulgar appear rather odd, fixed on him the character of a warlock. It was formerly believed and currently reported that he was in compact with the devil; and that the time he spent in study was spent in learning the black art and holding conversation with Old Nick.*

Napier, however, will be remembered for making one of the most important contributions to the advance of knowledge. It was through the use of logarithms that Kepler was able to reduce his observations and make his breakthrough which then in turn underpinned Newton's theory of gravitation. In the preface to the *Mirifici logarithmorum canonis descriptio*, quoted above, Napier says he hoped that his logarithms will save calculators much time and free them from the *slippery errors* of calculations. Laplace, 200 year later, agreed, saying that logarithms:-

*...by shortening the labours, doubled the life of the astronomer .*

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