

# Karl Theodor Wilhelm Weierstrass

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**Born: 31 Oct 1815 in Ostenfelde, Westphalia (now Germany)**

**Died: 19 Feb 1897 in Berlin, Germany**

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**Karl Weierstrass's** father, Wilhelm Weierstrass, was secretary to the mayor of Ostenfelde at the time of Karl's birth. Wilhelm Weierstrass was a well educated man who had a broad knowledge of the arts and of the sciences. He certainly was well capable of attaining higher positions than he did, and this attitude may have been one of the reasons that Karl Weierstrass's early career was in posts well below his outstanding ability. Weierstrass's mother was Theodora Vonderforst and Karl was the eldest of Theodora and Wilhelm's four children, none of whom married.

Wilhelm Weierstrass became a tax inspector when Karl was eight years old. This job involved him in only spending short periods in any one place so Karl frequently moved from school to school as the family moved around Prussia. In 1827 Karl's mother Theodora died and one year later his father Wilhelm remarried. By 1829 Wilhelm Weierstrass had become an assistant at the main tax office in Paderborn, and Karl entered the Catholic Gymnasium there. Weierstrass excelled at the Gymnasium despite having to take on a part-time job as a bookkeeper to help out the family finances.

While at the Gymnasium Weierstrass certainly reached a level of mathematical competence far beyond what would have been expected. He regularly read Crelle's *Journal* and gave mathematical tuition to one of his brothers. However Weierstrass's father wished him to study finance and so, after graduating from the Gymnasium in 1834, he entered the University of Bonn with a course planned out for him which included the study of law, finance and economics. With the career in the Prussian administration that was planned for him by his father, this was indeed a well designed course. However, Weierstrass suffered from the conflict of either obeying his father's wishes or studying the subject he loved, namely mathematics.

The result of the conflict which went on inside Weierstrass was that he did not attend either the mathematics lectures or the lectures of his planned course. He reacted to the conflict inside him by pretending that he did not care about his studies, and he spent four years of intensive fencing and drinking. As Biermann writes in [1]:-

*... the conflict between duty and inclination led to physical and mental strain. He tried, in vain, to overcome his problems by participating in carefree student life ...*

He did study mathematics on his own, however, reading Laplace's *Mécanique céleste* and then a work by Jacobi on elliptic functions. He came to understand the necessary methods in elliptic function theory by studying transcripts of lectures by Gudermann. In a letter to Lie, written nearly 50 years later, he explained how he came to make the definite decision to study mathematics despite his father's wishes around this time (see [1]):-

*... when I became aware of [a letter from Abel to Legendre] in Crelle's Journal during my student years, [it] was of the utmost importance. The immediate derivation of the form of the representation of the function given by Abel ..., from the differential equation defining this function, was the first mathematical task I set myself; and its fortunate solution made me determined to devote myself wholly to mathematics; I made this decision in my seventh semester ...*

Weierstrass had made a decision to become a mathematician but he was still supposed to be on a course studying public finance and administration. After his decision, he spent one further semester at the University of Bonn, his eighth semester ending in 1838, and having failed to study the subjects he was enrolled for he simply left the University without taking the examinations. Weierstrass's father was desperately upset by his son giving

up his studies. He was persuaded by a family friend, the president of the law courts at Paderborn, to allow Karl to study at the Theological and Philosophical Academy of Münster so that he could take the necessary examinations to become a secondary school teacher.

On 22 May 1839 Weierstrass enrolled at the Academy in Münster. Gudermann lectured in Münster and this was the reason that Weierstrass was so keen to study there. Weierstrass attended Gudermann's lectures on elliptic functions, some of the first lectures on this topic to be given, and Gudermann strongly encouraged Weierstrass in his mathematical studies. Leaving Münster in the autumn of 1839, Weierstrass studied for the teacher's examination which he registered for in March 1840. By this time, however, Weierstrass's father had moved jobs yet again, becoming director of a salt works in January 1840, and the family was now living in Westernkotten near Lippstadt on the Lippe River, west of Paderborn.

At Weierstrass's request he was given a question on the paper he received in May 1840 on the representation of elliptic functions and he presented his own important research as an answer. Gudermann assessed the paper and rated Weierstrass's contribution:-

*... of equal rank with the discoverers who were crowned with glory.*

When, in later life, Weierstrass learnt of Gudermann's comments he said that he would have published his results had he known. Weierstrass also commented on how generous Gudermann had been in his praise, particularly since he had been highly critical of Gudermann's methods.

By April 1841 Weierstrass had taken the necessary oral examinations and he began one year probation as a teacher at the Gymnasium in Münster. Although he did not publish any mathematics at this time, he wrote three short papers in 1841 and 1842 which are described in [3]:-

*The concepts on which Weierstrass based his theory of functions of a complex variable in later years after 1857 are found explicitly in his unpublished works written in Münster from 1841 through 1842, while still under the influence of Gudermann. The transformation of his conception of an analytic function from a differentiable function to a function expandable into a convergent power series was made during this early period of Weierstrass's mathematical activity.*

Weierstrass began his career as a qualified teacher of mathematics at the Pro-Gymnasium in Deutsch Krone in West Prussia (now Poland) in 1842 where he remained until he moved to the Collegium Hoseanum in Braunsberg in 1848. As a teacher of mathematics he was required to teach other topics too, and Weierstrass taught physics, botany, geography, history, German, calligraphy and even gymnastics. In later life Weierstrass described the "unending dreariness and boredom" of these miserable years in which [1]:-

*... he had neither a colleague for mathematical discussions nor access to a mathematical library, and that the exchange of scientific letters was a luxury that he could not afford.*

From around 1850 Weierstrass began to suffer from attacks of dizziness which were very severe and which ended after about an hour in violent sickness. Frequent attacks over a period of about 12 years made it difficult for him to work and it is thought that these problems may well have been caused by the mental conflicts he had suffered as a student, together with the stress of applying himself to mathematics in every free minute of his time while undertaking the demanding teaching job.

It is not surprising that when Weierstrass published papers on abelian functions in the Braunsberg school prospectus they went unnoticed by mathematicians. However, in 1854 he published *Zur Theorie der Abelschen Functionen* in Crelle's Journal and this was certainly noticed. This paper did not give the full theory of inversion of hyperelliptic integrals that Weierstrass had developed but rather gave a preliminary description of his methods involving representing abelian functions as constantly converging power series.

With this paper Weierstrass burst from obscurity. The University of Königsberg conferred an honorary doctor's degree on him on 31 March 1854. In 1855 Weierstrass applied for the chair at the University of Breslau left vacant when Kummer moved to Berlin. Kummer, however, tried to influence things so that Weierstrass would go to Berlin, not Breslau, so Weierstrass was not appointed. A letter from Dirichlet to the Prussian Minister of Culture written in 1855 strongly supported Weierstrass being given a university appointment. Details are given in [10]:-

*After being promoted to senior lecturer at Braunsberg, Weierstrass obtained a year's leave of absence to devote himself to advanced mathematical study. He had already decided, however, that he would never return to school teaching.*

Weierstrass published a full version of his theory of inversion of hyperelliptic integrals in his next paper *Theorie der Abelschen Functionen* in Crelle's Journal in 1856. There was a move from a number of universities to offer him a chair. While universities in Austria were discussing the prospect, an offer of a chair came from the Industry Institute in Berlin (later the Technische Hochschule). Although he would have preferred to go to the University of Berlin, Weierstrass certainly did not want to return to the Collegium Hosaeum in Braunsberg so he accepted the offer from the Institute on 14 June 1856.

Offers continued to be made to Weierstrass so that when he attended a conference in Vienna in September 1856 he was offered a chair at any Austrian university of his choice. Before he had decided what to do about this offer, the University of Berlin offered him a professorship in October. This was the job he had long wanted and he accepted quickly, although having accepted the offer from the Industry Institute earlier in the year he was not able to formally occupy the University of Berlin chair for some years.

Weierstrass's successful lectures in mathematics attracted students from all over the world. The topics of his lectures included:- the application of Fourier series and integrals to mathematical physics (1856/57), an introduction to the theory of analytic functions (where he set out results he had obtained in 1841 but never published), the theory of elliptic functions (his main research topic), and applications to problems in geometry and mechanics.

In his lectures of 1859/60 Weierstrass gave *Introduction to analysis* where he tackled the foundations of the subject for the first time. In 1860/61 he lectured on the *Integral calculus*. In 1861 his emphasis on rigour led him to discover a function that, although continuous, had no derivative at any point. Analysts who depended heavily upon intuition for their discoveries were rather dismayed at this counter-intuitive function.

We described above the health problems that Weierstrass suffered from 1850 onwards. Although he had achieved the positions that he had dreamed of, his health gave out in December 1861 when he collapsed completely. It took him about a year to recover sufficiently to lecture again and he was never to regain his health completely. From this time on he lectured sitting down while a student wrote on the blackboard for him. The attacks that he had suffered from 1850 stopped and were replaced by chest problems.

In his 1863/64 course on *The general theory of analytic functions* Weierstrass began to formulate his theory of the real numbers. In his 1863 lectures he proved that the complex numbers are the only commutative algebraic extension of the real numbers. Gauss had promised a proof of this in 1831 but had failed to give one.

Weierstrass's lectures developed into a four-semester course which he continued to give until 1890. The four courses were

1. *Introduction to the theory of analytic functions,*
2. *Elliptic functions,*
3. *Abelian functions,*
4. *Calculus of variations or applications of elliptic functions.*

Through the years the courses developed and a number of versions have been published such as the notes by Killing made in 1868 and those by Hurwitz from 1878. Weierstrass's approach still dominates teaching analysis today and this is clearly seen from the contents and style of these lectures, particularly the Introduction course. Its contents were: numbers, the function concept with Weierstrass's power series approach, continuity and differentiability, analytic continuation, points of singularity, analytic functions of several variables, in particular Weierstrass's "preparation theorem", and contour integrals.

At Berlin, Weierstrass had two colleagues Kummer and Kronecker and together the three gave Berlin a reputation as the leading university at which to study mathematics. Kronecker was a close friend of Weierstrass's for many years but in 1877 Kronecker's opposition to Cantor's work caused a rift between the two men. This became so bad that at one stage, in 1885, Weierstrass decided to leave Berlin and go to Switzerland. However, he changed his mind and remained in Berlin.

A large number of students benefited from Weierstrass's teaching. We name a few who are mentioned elsewhere in our archive: Bachmann, Bolza, Cantor, Engel, Frobenius, Gegenbauer, Hensel, Hölder, Hurwitz, Killing, Klein, Kneser, Königsberger, Lerch, Lie, Lueroth, Mertens, Minkowski, Mittag-Leffler, Netto, Schottky, Schwarz and Stolz. One student in particular, however, deserves special mention.

In 1870 Sofia Kovalevskaya came to Berlin and Weierstrass taught her privately since she was not allowed admission to the university. Clearly she was a very special student as far as Weierstrass was concerned for he wrote to her that he:-

*... dreamed and been enraptured of so many riddles that remain for us to solve, on finite and infinite spaces, on the stability of the world system, and on all the other major problems of the mathematics and the physics of the future. ... you have been close ... throughout my entire life ... and never have I found anyone who could bring me such understanding of the highest aims of science and such joyful accord with my intentions and basic principles as you.*

It was through Weierstrass's efforts that Kovalevskaya received an honorary doctorate from Göttingen, and he also used his influence to help her obtain the post in Stockholm in 1883. Weierstrass and Kovalevskaya corresponded for 20 years between 1871 to 1890. More than 160 letters were exchanged (see [5], [7] etc.), but Weierstrass burnt Kovalevskaya's letters after her death.

The standards of rigour that Weierstrass set, defining, for example, irrational numbers as limits of convergent series, strongly affected the future of mathematics. He also studied entire functions, the notion of uniform convergence and functions defined by infinite products. His efforts are summed up in [2] as follows:-

*Known as the father of modern analysis, Weierstrass devised tests for the convergence of series and contributed to the theory of periodic functions, functions of real variables, elliptic functions, Abelian functions, converging infinite products, and the calculus of variations. He also advanced the theory of bilinear and quadratic forms.*

Weierstrass published little [1]:-

*... because his critical sense invariably compelled him to base any analysis on a firm foundation, starting from a fresh approach and continually revising and expanding.*

However, he did edit the complete works of Steiner and those of Jacobi. He decided to supervise the publication of his own complete works, in his case this would involve a great deal of unpublished material from his lecture courses and Weierstrass realised that without his help this would be a difficult task. The first two volumes appeared in 1894 and 1895, being the only ones to appear before his death in 1897. His last years were difficult ones [1]:-

*During his last three years he was confined to a wheelchair, immobile and dependent. He died of pneumonia.*

The remaining volumes of his Complete Works appeared slowly; volume 3 in 1903, volume 4 in 1902, volumes 5 and 6 in 1915, and volume 7 in 1927. The seven volumes were reprinted in 1967. More work continues to be published today, particularly versions of his lecture courses taken from the notes made by those who attended the lectures.

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**October 1998**