Niels Henrik Abel

Born: 5 Aug 1802 in Frindoe (near Stavanger), Norway
Died: 6 April 1829 in Froland, Norway

Niels Abel's life was dominated by poverty and we begin by putting this in context by looking briefly at the political problems which led to economic problems in Norway. At the end of the 18th century Norway was part of Denmark and the Danish tried to remain neutral through the Napoleonic wars. However a neutrality treaty in 1794 was considered a aggressive act by England and, in 1801, the English fleet destroyed most of the Danish fleet in a battle in the harbour at Copenhagen. Despite this Denmark-Norway avoided wars until 1807 when England feared that the Danish fleet might be used by the French to invade. Using the philosophy that attack is the best form of defence, the English attacked and captured the whole Danish fleet in October 1807.

Denmark then joined the alliance against England. The continental powers blockaded England, and as a counter to this England blockaded Norway. The twin blockade was a catastrophe to Norway preventing their timber exports, which had been largely to Britain, and preventing their grain imports from Denmark. An economic crisis in Norway followed with the people suffering hunger and extreme poverty. In 1813 Sweden attacked Denmark from the south and, at the treaty of Kiel in January 1814, Denmark handed over Norway to Sweden. An attempt at independence by Norway a few months later led to Sweden attacking Norway in July 1814. Sweden gained control of Norway, setting up a complete internal self-government for Norway with a government in Christiania (which is called Oslo today). In this difficult time Abel was growing up in Gjerstad in south-east Norway.

Abel's father, Soren Georg Abel, had a degree in theology and philology and his father (Niels Abel's grandfather) was a Protestant minister at Gjerstad near Risor. Soren Abel was a Norwegian nationalist who was active politically in the movement to make Norway independent. Soren Abel married Ane Marie Simonson, the daughter of a merchant and ship owner, and was appointed as minister at Finnoy. Niels Abel, the second of seven children, was one year old when his grandfather died and his father was appointed to succeed him as the minister at Gjerstad. It was in that town that Abel was brought up, taught by his father in the vicarage until he reached 13 years of age. However, these were the 13 years of economic crisis for Norway described above and Abel's parents would have not been able to feed their family that well. The problems were not entirely political either for [14]:-

[*Abel's*] *father was probably a drunkard and his mother was accused of having lax morals.*

Abel's father was, however, important in the politics of Norway and, after Sweden gained control of Norway in 1814, he was involved in writing a new constitution for Norway as a member of the Storting, the Norwegian legislative body. In 1815 Abel and his older brother were sent to the Cathedral School in Christiania. The founding of the University of Christiania had taken away the good teachers from the Cathedral School to staff the University when it opened for teaching in 1813. What had been a good school was in a bad state when Abel arrived. Uninspired by the poor school, he proved a rather ordinary pupil with some talent for mathematics and physics.

When a new mathematics teacher Bernt Holmboe joined the school in 1817 things changed markedly for Abel. The previous mathematics teacher had been dismissed for punishing a boy so severely that he had died. Abel began to study university level mathematics texts and, within a year of Holmboe's arrival, Abel was reading the works of Euler, Newton, Lalande and d'Alembert. Holmboe was convinced that Abel had great talent and encouraged him greatly taking him on to study the works of Lagrange and Laplace. However, in 1820 tragedy struck Abel's family when his father died.

Abel's father had ended his political career in disgrace by making false charges against his colleagues in the Storting after he was elected to the body again in 1818. His habits of drinking to excess also contributed to his dismissal and the family was therefore in the deepest trouble when he died. There was now no money to allow Abel to complete his school education, nor money to allow him to study at university and, in addition, Abel had the responsibility of supporting his mother and family.

Holmboe was able to help Abel gain a scholarship to remain at school and Abel was able to enter the University of Christiania in 1821, ten years after the university was founded. Holmboe had raised money from his colleagues to enable Abel to study at the university and he graduated in 1822. While in his final year at school, however, Abel had begun working on the solution of quintic equations by radicals. He believed that he had solved the quintic in 1821 and submitted a paper to the Danish mathematician Ferdinand Degen, for publication by the Royal Society of Copenhagen. Degen asked Abel to give a numerical example of his method and, while trying to provide an example, Abel discovered the mistake in his paper. Degen had given Abel some important advice that was to set him working on an area of mathematics (see [2]):-

... whose development would have the greatest consequences for analysis and mechanics. I refer to elliptic integrals. A serious investigator with suitable qualifications for research of this kind would by no means be restricted to the many beautiful properties of these most remarkable functions, but could discover a Strait of Magellan leading into wide expanses of a tremendous analytic ocean.

At the University of Christiania Abel found a supporter in the professor of astronomy Christopher Hansteen, who provided both financial support and encouragement. Hansteen's wife began to care for Abel as if he was her own son. In 1823 Abel published papers on functional equations and integrals in a new scientific journal started up by Hansteen. In Abel's third paper, *Solutions of some problems by means of definite integrals* he gave the first solution of an integral equation.

Abel was given a small grant to visit Degen and other mathematicians in Copenhagen. While there he met Christine Kemp who shortly afterwards became his fiancée. Returning to Christiania, Abel tried to get the University of Christiania to give him a larger grant to enable him to visit the top mathematicians in Germany and France. He did not speak French of German so, partly to save money, he was given funds to remain in Christiania for two years to give him the chance to become fluent in these languages before travelling. Abel began working again on quintic equations and, in 1824, he proved the impossibility of solving the general equation of the fifth degree in radicals. He published the work in French and at his own expense since he wanted an impressive piece of work to take with him when he was on his travels. As Ayoub writes in [6]:-

He chose a pamphlet as the quickest way to get it into print, and in order to save on the printing costs, he reduced the proof to fit on half a folio sheet [*six pages*].

By this time Abel seems to have known something of Ruffini's work for he had studied Cauchy's work of 1815 while he was an undergraduate and in this paper there is a reference to Ruffini's work. Abel's 1824 paper begins ([6]):-

Geometers have occupied themselves a great deal with the general solution of algebraic equations and several among them have sought to prove the impossibility. But, if I am not mistaken, they have not succeeded up to the present.

Abel sent this pamphlet to several mathematicians including Gauss, who he intended to visit in Göttingen while on his travels. In August 1825 Abel was given a scholarship from the Norwegian government to allow him to travel abroad and, after taking a month to settle his affairs, he set out for the Continent with four friends, first visiting mathematicians in Norway and Denmark. On reaching Copenhagen, Abel found that Degen had died and he changed his mind about taking Hansteen's advice to go directly to Paris, preferring not to travel alone and stay with his friends who were going to Berlin. As he wrote in a later letter ([7]):-

Now I am so constituted that I cannot endure solitude. Alone, I am depressed, I get cantankerous, and I have little inclination to work.

In Copenhagen Abel was given a letter of introduction to Crelle by one of the mathematicians there. Abel met Crelle in Berlin and the two became firm friends. This proved the most useful part of Abel's whole trip, particularly as Crelle was about to begin publishing a journal devoted to mathematical research. Abel was encouraged by Crelle to write a clearer version of his work on the insolubility of the quintic and this resulted in *Recherches sur les fonctions elliptiques* which was published in 1827 in the first volume of *Crelle's Journal*, along with six other papers by Abel. While in Berlin, Abel learnt that the position of professor of mathematics at the University of Christiania, the only university in Norway, had been given to Holmboe. With no prospects of a university post in Norway, Abel began to worry about his future.

Crelle's Journal continued to be a source for Abel's papers and Abel began to work to establish mathematical analysis on a rigorous basis. He wrote to Holmboe from Berlin [2]:-

My eyes have been opened in the most surprising manner. If you disregard the very simplest cases, there is in all of mathematics not a single infinite series whose sum had been rigorously determined. In other words, the most important parts of mathematics stand without foundation. It is true that most of it is valid, but that is very surprising. I struggle to find a reason for it, an exceedingly interesting problem.

It had been Abel's intention to travel with Crelle to Paris and to visit Gauss in Göttingen on the way. However, news got back to Abel that Gauss was not pleased to receive his work on the insolubility of the quintic, so Abel decided that he would be better not to go to Göttingen. It is uncertain why Gauss took this attitude towards Abel's work since he certainly never read it - the paper was found unopened after Gauss's death. Ayoub gives two possible reasons [6]:-

... the first possibility is that Gauss *had proved the result himself and was willing to let Abel take the credit. ... The other explanation is that he did not attach very much importance to solvability by radicals...*

The second of these explanations does seem the more likely, especially since Gauss had written in his thesis of 1801 that the algebraic solution of an equation was no better than devising a symbol for the root of the equation and then saying that the equation had a root equal to the symbol.

Crelle was detained in Berlin and could not travel with Abel to Paris. Abel therefore did not go directly to Paris, but chose to travel again with his Norwegian friends to northern Italy before crossing the Alps to France. In Paris Abel was disappointed to find there was little interest in his work. He wrote back to Holmboe ([7]):-

The French are much more reserved with strangers than the Germans. It is extremely difficult to gain their intimacy, and I do not dare to urge my pretensions as far as that; finally every beginner had a great deal of difficulty getting noticed here. I have just finished an extensive treatise on a certain class of transcendental functions to present it to the Institute which will be done next Monday. I showed it to Mr Cauchy*, but he scarcely deigned to glance at it.*

The contents and importance of this treatise by Abel is described in [2]:-

It dealt with the sum of integrals of a given algebraic function. Abel's theorem states that any such sum can be expressed as a fixed number p of these integrals, with integration arguments that are algebraic functions of the original arguments. The minimal number p is the genus of the algebraic function, and this is the first occurrence of this fundamental quantity. Abel's theorem is a vast generalisation of Euler*'s relation for elliptic integrals.*

Two referees, Cauchy and Legendre, were appointed to referee the paper and Abel remained in Paris for a few months [14]:-

... emaciated, gloomy, weary and constantly worried. He ... could only afford to eat one meal a day.

He published some articles, mainly on the results he had already written for *Crelle's Journal*, then with no money left and his health in a very poor state, he returned to Berlin at the end of 1826. In Berlin, Abel borrowed some money and continued working on elliptic functions. He wrote a paper in which [2]:-

... he radically transformed the theory of elliptic integrals to the theory of elliptic functions by using their inverse functions ...

Crelle tried to persuade Abel to remain in Berlin until he could find an academic post for him and he even offered Abel the editorship of *Crelle's Journal*. However, Abel wanted to get home and by this time he was heavily in debt. He reached Christiania in May 1827 and was awarded a small amount of money by the university although they made sure they had the right to deduct a corresponding amount from any future salary he earned. To make a little more money Abel tutored schoolchildren and his fiancée was employed as a governess to friends of Abel's family in Froland.

Hansteen received a major grant to investigate the Earth's magnetic field in Siberia and a replacement was needed to teach for him at the University and also at the Military Academy. Abel was appointed to this post which improved his position a little.

In 1828 Abel was shown a paper by Jacobi on transformations of elliptic integrals. Abel quickly showed that Jacobi's results were consequences of his own and added a note to this effect to the second part of his major work on elliptic functions. He had been working again on the algebraic solution of equations, with the aim of solving the problem of which equations were soluble by radicals (the problem which Galois solved a few years later). He put this to one side to compete with Jacobi in the theory of elliptic functions, quickly writing several papers on the topic.

Legendre saw the new ideas in the papers which Abel and Jacobi were writing and said ([2]):-

Through these works you two will be placed in the class of the foremost analysts of our times.

Abel continued to pour out high quality mathematics as his health continued to deteriorate. He spent the summer vacation of 1828 with his fiancée in Froland. The masterpiece which he had submitted to the Paris Academy seemed to have been lost and so he wrote the main result down again [3]:-

The paper was only two brief pages, but of all his many works perhaps the most poignant. He called it only "A theorem": it had no introduction, contained no superfluous remarks, no applications. It was a monument resplendent in its simple lines - the main theorem from his Paris memoir, formulated in few words.

Abel travelled by sled to visit his fiancée again in Froland for Christmas 1828. He became seriously ill on the sled journey and despite an improvement which allowed them to enjoy Christmas, he soon became very seriously ill again. Crelle was told and he redoubled his efforts to obtain an appointment for Abel in Berlin. He succeeded and wrote to Abel on the 8 April 1829 to tell him the good news. It was too late, Abel had already died. Ore [3] describes his last few days:-

... the weakness and cough increased and he could remain out of bed only the few minutes while it was being made. Occasionally he would attempt to work on his mathematics, but he could no longer write. Sometimes he lived in the past, talking about his poverty and about Fru Hansteen's goodness. Always he was kind and patient. ...

He endured his worst agony during the night of April 5*. Towards morning he became more quiet and in the forenoon, at eleven o'clock, he expired his last sigh.*

After Abel's death his Paris memoir was found by Cauchy in 1830 after much searching. It was printed in 1841 but rather remarkably vanished again and was not found until 1952 when it turned up in Florence. Also after Abel's death unpublished work on the algebraic solution of equations was found. In fact in a letter Abel had written to Crelle on 18 October 1828 he gave the theorem [13]:-

If every three roots of an irreducible equation of prime degree are related to one another in such a way that one of them may be expressed rationally in terms of the other two, then the equation is soluble in radicals.

This result is essentially identical to one given by Galois in his famous memoir of 1830. In this same year 1830 the Paris Academy awarded Abel and Jacobi the Grand Prix for their outstanding work.

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