Apollonius of Perga

Born: about 262 BC in Perga, Pamphylia, Greek Ionia (now Murtina, Antalya, Turkey)
Died: about 190 BC in Alexandria, Egypt

Apollonius of Perga was known as 'The Great Geometer'. Little is known of his life but his works have had a very great influence on the development of mathematics, in particular his famous book *Conics* introduced terms which are familiar to us today such as parabola, ellipse and hyperbola.

Apollonius of Perga should not be confused with other Greek scholars called Apollonius, for it was a common name. In [1] details of others with the name of Apollonius are given: Apollonius of Rhodes, born about 295 BC, a Greek poet and grammarian, a pupil of Callimachus who was a teacher of Eratosthenes; Apollonius of Tralles, 2nd century BC, a Greek sculptor; Apollonius the Athenian, 1st century BC, a sculptor; Apollonius of Tyana, 1st century AD, a member of the society founded by Pythagoras; Apollonius Dyscolus, 2nd century AD, a Greek grammarian who was reputedly the founder of the systematic study of grammar; and Apollonius of Tyre who is a literary character.

The mathematician Apollonius was born in Perga, Pamphylia which today is known as Murtina, or Murtana and is now in Antalya, Turkey. Perga was a centre of culture at this time and it was the place of worship of Queen Artemis, a nature goddess. When he was a young man Apollonius went to Alexandria where he studied under the followers of Euclid and later he taught there. Apollonius visited Pergamum where a university and library similar to Alexandria had been built. Pergamum, today the town of Bergama in the province of Izmir in Turkey, was an ancient Greek city in Mysia. It was situated 25 km from the Aegean Sea on a hill on the northern side of the wide valley of the Caicus River (called the Bakir river today).

While Apollonius was at Pergamum he met Eudemus of Pergamum (not to be confused with Eudemus of Rhodes who wrote the *History of Geometry*) and also Attalus, who many think must be King Attalus I of Pergamum. In the preface to the second edition of *Conics* Apollonius addressed Eudemus (see [4] or [5]):-

If you are in good health and things are in other respects as you wish, it is well; with me too things are moderately well. During the time I spent with you at Pergamum I observed your eagerness to become aquatinted with my work in conics.

The only other pieces of information about Apollonius's life is to be found in the prefaces of various books of *Conics.* We learn that he had a son, also called Apollonius, and in fact his son took the second edition of book two of *Conics* from Alexandria to Eudemus in Pergamum. We also learn from the preface to this book that Apollonius introduced the geometer Philonides to Eudemus while they were at Ephesus.

We are in a somewhat better state of knowledge concerning the books which Apollonius wrote. *Conics* was written in eight books but only the first four have survived in Greek. In Arabic, however, the first seven of the eight books of *Conics* survive.

First we should note that conic sections to Apollonius are by definition the curves formed when a plane intersects the surface of a cone. Apollonius explains in his preface how he came to write his famous work *Conics* (see [4] or [5]):-

... I undertook the investigation of this subject at the request of Naucrates the geometer, at the time when he came to Alexandria and stayed with me, and, when I had worked it out in eight books, I gave them to him at

once, too hurriedly, because he was on the point of sailing; they had therefore not been thoroughly revised, indeed I had put down everything just as it occurred to me, postponing revision until the end.

Books 1 and 2 of the *Conics* began to circulate in the form of their first draft, in fact there is some evidence that certain translations which have come down to us have come from these first drafts. Apollonius writes (see [4] or [5]):-

... it happened that some persons also, among those who I have met, have got the first and second books before they were corrected....

Conics consisted of 8 books. Books one to four form an elementary introduction to the basic properties of conics. Most of the results in these books were known to Euclid, Aristaeus and others but some are, in Apollonius's own words:-

... worked out more fully and generally than in the writings of others.

In book one the relations satisfied by the diameters and tangents of conics are studied while in book two Apollonius investigates how hyperbolas are related to their asymptotes, and he also studies how to draw tangents to given conics. There are, however, new results in these books in particular in book three. Apollonius writes of book three (see [4] or [5]):-

... the most and prettiest of these theorems are new, and it was their discovery which made me aware that Euclid did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me.

Books five to seven are highly original. In these Apollonius discusses normals to conics and shows how many can be drawn from a point. He gives propositions determining the centre of curvature which lead immediately to the Cartesian equation of the evolute. Heath writes that book five [5]:-

... is the most remarkable of the extant Books. It deals with normals to conics regarded as maximum and minimum straight lines drawn from particular points to the curve. Included in it are a series of propositions which, though worked out by the purest geometrical methods, actually lead immediately to the determination of the evolute of each of the three conics; that is to say, the Cartesian equations of the evolutes can be easily deduced from the results obtained by Apollonius. There can be no doubt that the Book is almost wholly original, and it is a veritable geometrical tour de force.

The beauty of Apollonius's *Conics* can readily be seen by reading the propositions as given by Heath, see [4] or [5]. However, Heath explains in [5] how difficult the original text is to read:-

... the treatise is a great classic which deserves to be more known than it is. What militates against its being read in its original form is the great extent of the exposition (*it contains* 387 *separate propositions*)*, due partly to the Greek habit of proving particular cases of a general proposition separately from the proposition itself, but more to the cumbersomeness of the enunciations of complicated propositions in general terms* (*without the help of letters to denote particular points*) *and to the elaborateness of the Euclidean form, to which Apollonius adheres throughout.*

Pappus gives some indications of the contents of six other works by Apollonius. These are *Cutting of a ratio* (in two books), *Cutting an area* (in two books), *On determinate section* (in two books), *Tangencies* (in two books), *Plane loci* (in two books), and *On verging constructions* (in two books). *Cutting of a ratio* survives in Arabic and we are told by the 10^{th} century bibliographer Ibn al-Nadim that three other works were translated into Arabic but none of these survives.

To illustrate how far Apollonius had taken geometric constructions beyond that of Euclid's *Elements* we consider results which are known to have been contained in *Tangencies.* In the *Elements* Book III Euclid shows how to draw a circle through three given points. He also shows how to draw a tangent to three given lines. In *Tangencies* Apollonius shows how to construct the circle which is tangent to three given circles. More generally he shows how to construct the circle which is tangent to any three objects, where the objects are points or lines or circles.

In [11] Hogendijk reports that two works of Apollonius, not previously thought to have been translated into Arabic, were in fact known to Muslim geometers of the 10^{th} century. These are the works *Plane loci* and *On verging constructions.* In [11] some results from these works which were not previously known to have been proved by Apollonius are described.

From other sources there are references to still further books by Apollonius, none of which have survived. Hypsicles refers to a work by Apollonius comparing a dodecahedron and an icosahedron inscribed in the same sphere, which like *Conics* appeared in two editions. Marinus, writing a commentary on Euclid's *Data*, refers to a general work by Apollonius in which the foundations of mathematics such as the meaning of axioms and definitions are discussed. Apollonius also wrote a work on the cylindrical helix and another on irrational numbers which is mentioned by Proclus. Eutocius refers to a book *Quick delivery* by Apollonius in which he obtained an approximation for π better than the

$$^{223}/_{71} < \pi < {}^{22}/_{7}$$

known to Archimedes. In *On the Burning Mirror* Apollonius showed that parallel rays of light are not brought to a focus by a spherical mirror (as had been previously thought) and discussed the focal properties of a parabolic mirror.

Apollonius was also an important founder of Greek mathematical astronomy, which used geometrical models to explain planetary theory. Ptolemy in his book *Syntaxis* says Apollonius introduced systems of eccentric and epicyclic motion to explain the apparent motion of the planets across the sky. This is not strictly true since the theory of epicycles certainly predates Apollonius. Nevertheless, Apollonius did make substantial contributions particularly using his great geometric skills. In particular, he made a study of the points where a planet appears stationary, namely the points where the forward motion changes to a retrograde motion or the converse.

There were also applications made by Apollonius, using his knowledge of conics, to practical problems. He developed the hemicyclium, a sundial which has the hour lines drawn on the surface of a conic section giving greater accuracy.

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