Alexis Claude Clairaut

Born: 7 May 1713 in Paris, France
Died: 17 May 1765 in Paris, France

Alexis Clairaut's father, Jean-Baptiste Clairaut, taught mathematics in Paris and showed his quality by being elected to the Berlin Academy. Alexis's mother, Catherine Petit, had twenty children although only Alexis survived to adulthood.

Jean-Baptiste Clairaut educated his son at home and set unbelievably high standards. Alexis used Euclid's *Elements* while learning to read and by the age of nine he had mastered the excellent mathematics textbook of Guisnée *Application de l'algèbre à la géométrie* which provided a good introduction to the differential and integral calculus as well as analytical geometry. In the following year, Clairaut went on to study de L'Hôpital's books, in particular his famous text *Analyse des infiniment petits pour l'intelligence des lignes courbes.*

Few people have read their first paper to an academy at the age of 13, but this was the incredible achievement of Clairaut's in 1726 when he read his paper *Quatre problèmes sur de nouvelles courbes* to the Paris Academy. Although we have already noted that Clairaut was the only one of twenty children of his parents to reach adulthood, he did have a younger brother who, at the age of 14, read a mathematics paper to the Academy in 1730. This younger brother died in 1732 at the age of 16.

Clairaut began to undertake research on double curvature curves which he completed in 1729. As a result of this work he was proposed for membership of the Paris Academy on 4 September 1729 but the king did not confirm his election until 1731. In July 1731 Clairaut became the youngest person ever elected to the Paris Academy of Sciences. There he joined a small group, led by Pierre Louis Maupertuis, who supported the natural philosophy of Newton. Maupertuis was 15 years older than Clairaut but despite this, at the age of 33, he was also a young member of the Academy.

Clairaut became close friends of Maupertuis, Voltaire, and du Châtelet. This was much more than a personal friendship since he did important work with both Maupertuis and du Châtelet. He helped the Marquise du Châtelet translate Newton's *Principia* into French, a project which began before 1745 and continued until part of the book was published in 1756. Many of Clairaut's own theories were added to the book, in addition to the translation of Newton by du Châtelet.

Together with Maupertuis, Clairaut visited Basel in 1734 to spend a few months studying with Johann Bernoulli. While in Basel, Clairaut became friends with Samuel König and, for many years, the two continued a useful scientific collaboration by correspondence.

Clairaut published some important work during the period 1733 to 1743. He wrote the paper *Sur quelques questions de maximis et minimis* in 1733 on the calculus of variations, written in the style of Johann Bernoulli and, in the same year, he published on the geodesics of quadrics of rotation again studying a topic to which Johann Bernoulli had contributed. The following year Clairaut studied the differential equations now known as 'Clairaut's differential equations' and gave a singular solution in addition to the general integral of the equations. In 1739 and 1740 he published further work on the integral calculus, proving the existence of integrating factors for solving first order differential equations (a topic which also interested Johann Bernoulli, Reyneau and Euler). In 1742 Clairaut published an important work on dynamics but, in the following year, he turned his attention to the topic for which he is best known. He became interested in solving theoretical questions which followed on from the practical results of an expedition some years earlier.

From 20 April 1736 to 20 August 1737 Clairaut had taken part in an expedition to Lapland, led by Maupertuis, to measure a degree of longitude. The expedition was organised by the Paris Academy of Sciences, still continuing the programme started by Cassini, to verify Newton's theoretical proof that the Earth is an oblate spheroid. In addition to Maupertuis and Clairaut, the group contained other young scientists such as Lemonnier, Camus and Celsius. The highly successful team were not without their critics [1]:-

This enthusiastic group accomplished its mission quickly and precisely, in an atmosphere of youthful gaiety for which some reproached them.

In 1743 Clairaut published *Théorie de la figure de la Terre* confirming the Newton-Huygens belief that the Earth was flattened at the poles. The book was a theoretical study to support the experimental data on the shape of the Earth which the expedition to Lapland had gathered. The book was an important one in laying the foundations for the study of hydrostatics. It built on foundations due to Newton and Huygens who had put forward the theory that the Earth was an oblate spheroid, and also on Maclaurin's work on tides which developed some background results in hydrostatics.

After his work on *Théorie de la figure de la Terre* Clairaut began to work on the three-body problem in 1745, in particular on the problem of the moon's orbit. The first conclusions that he drew from his work was that Newton's theory of gravity was incorrect and that the inverse square law did not hold. In this Clairaut had the support of Euler who, after learning of Clairaut's conclusions, wrote to him on 30 September 1747:-

I am able to give several proof that the forces which act on the moon do not exactly follow the rule of Newton, and the one you draw from the movement of the apogee is the most striking...

Clairaut, more confident with Euler's support, announced to the Paris Academy on 15 November 1747 that the inverse square law was false. Rather remarkably, just before Clairaut made his announcement, d'Alembert deposited a paper with the Academy which showed that his calculations agreed with those of Clairaut. Clairaut suggested that a term in $1/r^4$ needed to be added and Euler (perhaps rather wisely) agreed that Clairaut had found the error in the inverse square law before he had.

Of course, not all mathematicians at this time believed Newton's theory, some still believing in Descartes' vortex theories. The announcement that Newton's law was incorrect made many of Descartes' supporters overjoyed and even Euler returned to Descartes' views. Some attacked Clairaut's announcement, for example Buffon who used a metaphysical argument based on the simplicity of the inverse square law.

However, by the spring of 1748, Clairaut realised that the difference between the observed motion of the moon's apogee and the one predicted by the theory was due to errors coming from the approximations that were being made rather than from the inverse square law of gravitational attraction. Clairaut announced to the Academy on 17 May 1749 that his theory was now in agreement with the inverse square law. He then had a period of enjoying watching d'Alembert and Euler struggle to repeat his calculations. Clairaut wrote to his friend Gabriel Cramer [14]:-

... d'Alembert and Euler had no inkling of the stratagem that led me to my new results. The latter twice wrote to tell me that he had made fruitless efforts to find the same thing as I, and that he begged me to tell him how I arrived at them. I told him, more or less, what it was all about...

Euler still felt he did not properly understand what Clairaut had done so he tried to tempt him to write it up properly by having the St Petersburg Academy to set the problem of the moon's apogee as the prize topic for 1752. Indeed his ploy worked and Clairaut submitted an essay which let Euler fully understand Clairaut's method. Euler, going well beyond the mark but showing how frustrated he had been not solving the problem himself, wrote to Clairaut that his results were:-

... the most important and profound discovery that has ever been made in mathematics.

Clairaut published *Théorie de la lune* in 1752 and this work, together with his lunar tables published two years later, completed his work on this particular problem.

Clairaut decided to apply his knowledge of the three-body problem to compute the orbit of Halley's comet and so predict the exact date of its return. This required much more accurate approximations than had the problem of the moon. He calculated to within a month the return in 1759 of Halley's comet to its perihelion (closest point to the Sun). He announced his result, that the perihelion would occur on 15 April 1759, to the Paris Academy on 14 November 1758, while the actual date of perihelion turned out to be 13 March. When the comet appeared, only one month before the predicted date, Clairaut was given great public acclaim. There was a suggestion that the comet be renamed after Clairaut, and Clairaut was called the 'new Thales'.

Clairaut improved his results when he used a different method in his prize winning paper submitted to the St Petersburg Academy for the 1762 prize. He was able to obtain the date of 30 March in this work which, given the complexity of the problem of taking the perturbations of Jupiter and Saturn into account, is remarkably good.

A dispute arose between Clairaut and d'Alembert regarding this work on comets. Although the two had been reasonably friendly rivals up to about 1747, after that relations deteriorated. When Clairaut wrote a review of d'Alembert's book containing lunar tables then, as Hankins writes [4]:-

He was not openly hostile, but adopted the condescending tone of a master instructing an able student; he praised d'Alembert's great analytical skill, but said his tables were of little use - at least compared to Clairaut's own tables.

In an attack on those who, like d'Alembert, concentrated on theory and neglected experiment, Clairaut wrote:-

In order to avoid delicate experiments or long tedious calculations, in order to substitute analytical methods which cost them less trouble, they often make hypotheses which have no place in nature; they pursue theories that are foreign to their object, whereas a little constancy in the execution of a perfectly simple method would have surely brought them to their goal.

When d'Alembert attacked Clairaut's solution of the three-body problem as being too much based on observation and not, like his own work, based on theoretical results, Clairaut strongly attacked d'Alembert in the most bitter dispute of their lives. It is hard to judge which of the two great mathematicians was right, but Clairaut clearly won the public argument at the time, not least because his standing was so high after the remarkable prediction of the date of the return of Halley's comet.

We should also mention another topic to which Clairaut made important contributions, namely to the aberration of light. He had to have a thorough understanding of this topic from the time of the observations made by the Lapland expedition. He also had to make use of corrections due to aberration in his work on the planets and comets. He was particularly interested in the ideas of improving telescope design by using lenses made up of two different types of glass. Clairaut wrote some important memoirs on the topic, studying the theory as well as conducting optical experiments. This work was still incomplete at the time of his death.

Clairaut worked on a wide range of problems within mathematics. A book on algebra *Elements d'algèbre* was published in 1749 and a geometry book *Elements de géometrie* in the year of his death 1765. In the preface to *Elements de géometrie* Clairaut gives his aims in writing the book:-

I intended to go back to what might have given rise to geometry; and I attempted to develop its principles by a method natural enough so that one might assume it to be the same as that of geometry's first inventors, attempting only to avoid any false steps that they might have had to take...

The algebra book was an even more scholarly work and took the subject up to the solution of equations of degree four. He tried, with great success, to show why the introduction of algebraic notation was necessary and inevitable. The book was used for teaching in French schools for many years.

Clairaut died at the age of 52 after a brief illness. He was at the height of his powers and he had been honoured by being elected to the leading academies of the day. He had been elected to the Royal Society of London, the Academy of Berlin, the Academy of St Petersburg and the Academies of Bologna and Uppsala.

Article by: *J J O'Connor* and *E F Robertson*

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