Marius Sophus Lie

Born: 17 Dec 1842 in Nordfjordeide, Norway
Died: 18 Feb 1899 in Kristiania (now Oslo), Norway

Sophus Lie's father was Johann Herman Lie, a Lutheran minister. His parents had six children and Sophus was the youngest of the six. Sophus first attended school in the town of Moss, which is a port in south-eastern Norway, on the eastern side of the Oslo Fjord. In 1857 he entered Nissen's Private Latin School in Christiania (the city which became Kristiania, then Oslo in 1925) . While at this school he decided to take up a military career, but his eyesight was not sufficiently good so he gave up the idea and entered University of Christiania.

At university Lie studied a broad science course. There was certainly some mathematics in this course, and Lie attended lectures by Sylow in 1862. Although not on the permanent staff, Sylow taught a course, substituting for Broch, in which he explained Abel's and Galois' work on algebraic equations. Lie also attended lectures by Carl Bjerknes on mathematics, so he certainly had teachers of considerable quality, yet he graduated in 1865 without having shown any great ability for the subject, or any great liking for it.

There followed a period when Lie could not decide what subject to pursue and he taught pupils while trying to make his decision. The one thing he knew he wanted was an academic career and he thought for a while that astronomy might be the right topic. He learnt some mechanics, wondered whether botany or zoology or physics might be the right subjects and in general became rather confused. However, there are signs that from 1866 he began to read more and more mathematics and the library records in the University of Christiania show clearly that his interests were steadily turning in that direction.

It was during the year 1867 that Lie had his first brilliant new mathematical idea. It came to him in the middle of the night and, filled with excitement, he rushed to see his friend Ernst Motzfeldt, woke him up and shouted:-

I have found it, it is quite simple!

This was not the end of Lie's problems of course (far from it for Lie would always have problems), but at least in his own mind he now knew the career he wanted and it would be fair to say that from that moment on Lie became a mathematician. The type of mathematics that Lie would study became more clearly defined during 1868 when he avidly read papers on geometry by Plücker and Poncelet. Plücker's [1]:-

... monumental idea to create new geometries by choosing figures other than points - in fact straight lines - as elements of space pervaded all of Lie's work.

Lie wrote a short mathematical paper in 1869, which he published at his own expense, based on the inspiration which had struck him in 1867. He wrote up a more detailed exposition, but the world of mathematics was too cautious to quickly accept Lie's revolutionary notions. The Academy of Science in Christiania was reluctant to publish his work, and at this stage Lie began to despair that he would become accepted in the mathematical world. His friend Motzfeldt did a superb job of encouraging Lie to press on with his mathematical ideas and the breakthrough came later in 1869 when *Crelle's Journal* accepted his paper. He sent letters to two Prussian mathematicians, Reye and Clebsch, still attempting to gain recognition for his ideas. The paper in *Crelle's Journal*, however, proved vital for, on the strength of the paper, Lie was awarded a scholarship to travel and meet the leading mathematicians.

Setting off near the end of the year 1869, Lie went to Prussia and visited Göttingen and then Berlin. In Berlin he met Kronecker, Kummer and Weierstrass. Lie was not attracted to the style of Weierstrass's mathematics which

dominated Berlin. His interests fitted more closely with Kummer, and Lie lectured on his own results in Kummer's seminar and was able to correct some errors that Kummer had made in his work on line congruences of degree 3. Most important to Lie, however, was the fact that in Berlin he met Felix Klein. It was easy to see that these two would instantly find common ground in mathematics since Klein had been a student of Plücker, and Lie, although he never met Plücker, always said that he felt like Plücker's student. Despite the common link through Plücker's line geometry, Lie and Klein were rather different in character as Freudenthal points out in [1]:-

Lie and Klein had quite different characters as humans and mathematicians: the algebraist Klein was fascinated by the peculiarities of charming problems; the analyst Lie, parting from special cases, sought to understand a problem in its appropriate generalisation.

It was in Berlin that Lie developed a new self-confidence in his mathematical ability. He received high praise from Kummer, and he received replies from Reye and Clebsch to his earlier letters which greatly encouraged him. Lie wrote to his friend Motzfeldt in Christiania saying (see for example [34]):-

... in the years 1864-68, *I really underestimated my own mental power.*

In the spring of 1870 Lie and Klein were together again in Paris. There they met Darboux, Chasles and Camille Jordan. Jordan seems to have succeeded in a way that Sylow did not, for Jordan made Lie realise how important group theory was for the study of geometry. Lie started to develop ideas which would later appear in his work on transformation groups. He began to discuss with Klein these new ideas on groups and geometry and he would collaborate later with Klein in publishing several papers. This joint work had as one of its outcomes Klein's characterisation of geometry in his Erlangen Program of 1872 as properties invariant under a group action. While in Paris Lie discovered contact transformations. These transformations allowed a 1-1 correspondence between lines and spheres in such a way that tangent spheres correspond to intersecting lines.

While Lie and Klein thought deeply about mathematics in Paris, the political situation between France and Prussia was deteriorating. The popularity of Napoleon III, the French emperor, was declining in France and he thought a war with Prussia might change his political fortunes since his advisers having told him that the French Army could defeat Prussia. Bismarck, the Prussian chancellor, saw a war with France as an opportunity to unite the South German states. With both sides feeling that a war was to their advantage, the Franco-Prussian War became inevitable. On 14 July, Bismarck sent a telegram which infuriated the French government and on the 19 July France declared war on Prussia. For Klein, a Prussian citizen who happened to be in Paris when war was declared, there was only one possibility: he had to return quickly to Berlin.

However, Lie was a Norwegian and he was finding mathematical discussions in Paris very stimulating. He decided to remain but became anxious as the German offensive met with only an ineffective French reply. In August, the German army trapped part of the French army in Metz and Lie decided it was time for him to leave and he planned to hike to Italy. He reached Fontainebleau but there he was arrested as a German spy, his mathematics notes being assumed to be top secret coded messages. Only after the intervention of Darboux was Lie released from prison. The French army had surrendered on 1 September, and on 19 September the German army began to blockade Paris. Lie fled again to Italy, then from there he made his way back to Christiania via Germany so that he could meet and discuss mathematics with Klein.

In 1871 Lie became an assistant at Christiania, having obtained a scholarship, and he also taught at Nissen's Private Latin School in Christiania where he had been a pupil himself. He submitted a dissertation *On a class of geometric transformations* (written in Norwegian) for his doctorate which was duly awarded in July 1872. The dissertation contained ideas from his first results published in *Crelle's Journal* and also the work on contact transformations, a special case of these transformations being a transformation which maps a line into a sphere, which he had discovered while in Paris. It was clear that Lie was a remarkable mathematician and the University of Christiania reacted in a very positive way, creating a chair for him in 1872. The famous Norwegian mathematician Abel had died more than 40 years before this (some 14 years before Lie was born)

but, despite Abel's short career, his complete works had not been published at that time. It was natural that Norwegian mathematicians would undertake the task, and between 1873 and 1881 Sylow and Lie prepared an edition of Abel's complete works. Lie, however, always claimed that most of the work was done by Sylow. Another event which took place within two years of Lie being appointed to his chair was his marriage. He married Anna Birch and they would have three children, one daughter and two sons.

Lie had started examining partial differential equations, hoping that he could find a theory which was analogous to the Galois theory of equations. He wrote:-

... the theory of differential equations is the most important discipline in modern mathematics.

He examined his contact transformations considering how they affected a process due to Jacobi of generating further solutions of differential equations from a given one. This led to combining the transformations in a way that Lie called an *infinitesimal group*, but which is not a group with our definition, rather what is today called a Lie algebra. It was during the winter of 1873-74 that Lie began to develop systematically what became his theory of continuous transformation groups, later called Lie groups leaving behind his original intention of examining partial differential equations. Later Killing was to examine the Lie algebras associated with Lie groups. He did this quite independently of Lie (and not it would appear in a manner which Lie found satisfactory), and it was Cartan who completed the classification of semisimple Lie algebras in 1900.

Although Lie was producing highly innovative mathematics, he became increasingly sad at the lack of recognition he was receiving in the mathematical world. One reason was undoubtedly his isolation in Christiania, but a second reason was that his papers were not easily understood, partly through his style of writing and partly because his geometrical intuition greatly exceeded that of other mathematicians. Klein, realising the problems, had the excellent idea of sending Friedrich Engel to Christiania to help Lie.

Engel had received his doctorate from Leipzig in 1883 having studied under Adolph Mayer writing a thesis on contact transformations. Klein recognised that he was the right man to assist Lie and, at Klein's suggestion, Engel went to work with Lie in Christiania starting in 1884. He worked with Lie for nine months leaving in 1885. Engel then was appointed to Leipzig and, when Klein left the chair at Leipzig in 1886, Lie was appointed to succeed him. The collaboration between Engel and Lie continued for nine years culminating with their joint major publication *Theorie der Transformationsgruppen* in three volumes between 1888 and 1893. This was Lie's major work on continuous groups of transformations. In Leipzig, life for Lie was rather different from that in Christiania. He was now in the mainstream of mathematics and students came from many countries to study under him. He had a much heavier teaching load, however [38]:-

Lie's lectures on his own research were highly rated by the students, in contrast to his somewhat unpopular obligatory lectures on standard topics. ... he preferred to draw a picture instead of giving rigorous proofs.

However all was not well, he still felt unrecognised and, as Svare writes in [38]:-

In Leipzig Lie was troubled by constant homesickness. A keen outdoor man, he missed the forests and mountains of Norway.

Towards the end of the 1880s Lie's relationship with Engel broke down. In 1892 the lifelong friendship between Lie and Klein broke down and the following year Lie publicly attacked Klein saying:-

I am no pupil of Klein, nor is the opposite the case, although this might be closer to the truth.

It is difficult for any biographer to represent these events, and the events which followed, fairly since there is a great deal of contradictory material in the literature. The reason for this is not hard to understand, for information about Lie was for many years based on [13] which Engel wrote on Lie's death. The position is complicated by the mental difficulties which Lie suffered in 1889. Klein's [34]:-

... "defence" of Lie's behaviour by referring to the close relationship between genius and madness really created a generally accepted explanation which has survived up to the present. By this act of "defence" Klein did his old friend an incredible injustice.

The truth is that Lie's behaviour was not totally irrational as it has been portrayed, but was indeed motivated by the way that both Engel and Klein had behaved. Purkert in [26] discusses the breakdown of relations between Lie and Engel. He has studied material from the University of Leipzig and believes that Lie changed his attitude toward Engel because Lie still felt a lack of recognition yet he knew that he was in a different class as a creative mathematician to Engel. Lie returned to Christiania in 1898 to take up a post specially created for him. He produced a report about who should fill his chair, and this is given in full in [26]. Despite Engel being one of the leading workers in Lie's own research field, Purkert believes that Lie's assessment that he lacked creativity was entirely fair.

In [15] Fritzsche comments on Lie's illness. He writes:-

Through information about Sophus Lie's illness it is possible to trace consequences that shed light on certain biographical aspects of his life; for example, his break with Friedrich Engel and Felix Klein. Furthermore, this evidence contradicts the oft-stated opinion that Lie's sickness was brought about by overwork.

Straume in [34] points out why Lie's behaviour towards Klein, with the final breakdown in 1892, was not irrational:-

Klein's Erlangen Program from 1872 *had not attracted much attention; in fact, it was Lie rather than Klein himself who had influenced the mathematical development envisioned in this Program. ... Klein decided to republish the Program and also write about its origins* (*in which Lie was much involved*)*, but Lie disagreed strongly with Klein's views on what had happened in the past. It also turned out that Klein burned all the letters he had received from Lie up to* 1877 (*and thus breaking a previous mutual agreement between them*).

Lie reacted by publicly attacking Klein in the Preface to the third volume of his *Theorie der Transformationsgruppen* in 1893. Certainly Lie was an angry man but he was attacking someone holding such a leading role on the world scene of mathematics that the attack was always more likely to rebound on Lie rather than hurt Klein. Already current research is showing Lie in a much better light over this affair (and therefore Klein in a less good one) than previously reported and all the indications are that further research will prove even more favourable to Lie.

Perhaps an indication of Lie's love for his homeland is the fact that he continued to hold his chair in Christiania from his first appointment in 1872, being officially on leave while holding the chair in Leipzig. However his health was already deteriorating when he returned to a chair in Christiania in 1898, and he died of pernicious anaemia in February 1899 soon after taking up the post.

Let us end by quoting from Robert Hermann's preface to [4]:-

In reading Lie's work in preparation for my commentary on these translations, I was overwhelmed by the richness and beauty of the geometric ideas flowing from Lie's work. Only a small part of this has been absorbed into mainstream mathematics. He thought and wrote in grandiose terms, in a style that has now gone out of fashion, and that would be censored by our scientific journals! The papers translated here and in the succeeding volumes of our translations present Lie in his wildest and greatest form.

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