George David Birkhoff

Born: 21 March 1884 in Overisel, Michigan, USA
Died: 12 Nov 1944 in Cambridge, Massachusetts, USA

George Birkhoff's parents were David Birkhoff, who was a medical doctor, and Jane Gertrude Droppers. George was educated in Chicago where he was a student at the Lewis Institute from 1896 to 1902. In 1901 he began a correspondence with Harry Vandiver on a problem in number theory. Together they studied the prime factors of $a^n - b^n$ and later wrote up their work for publication. Graduating from the Lewis Institute in 1902, Birkhoff began his university education.

He entered the University of Chicago in 1902, spending a year there, then moved to Harvard University where he studied from 1903 to 1905. While at Harvard he submitted the results he had obtained with Vandiver to the *Annals of Mathematics* in 1904 and this joint number theory paper became his first publication. Also in 1904 he submitted a paper on analysis to the American Mathematical Society. During these two years at Harvard the teacher who influenced him most was Bôcher who taught him algebra and classical analysis. Birkhoff was awarded his A.B. by Harvard in 1905 and his A.M. in 1906.

Birkhoff returned to the University of Chicago in 1905 to study for his doctorate. His research concentrated on asymptotic expansions, boundary value problems, and Sturm-Liouville type problems but his thesis advisor Eliakim Moore appears to have been a less influential guide to Birkhoff than was Poincaré. Birkhoff read Poincaré's works on differential equations and celestial mechanics and he learnt more, and was more strongly influenced in the direction his research was taking, by Poincaré than from his supervisor. Archibald writes in [5]:-

During his two years at Harvard, one in college and one in the graduate school, Bôcher and Osgood were in their prime; but during the next two years [*at Chicago*] *he worked under Eliakim Moore, Bolza and Maschke, in the most inspiring mathematical center in the United States at that time. His dissertation was done quite independently however.*

The doctoral thesis which Birkhoff submitted was entitled *Asymptotic Properties of Certain Ordinary Differential Equations with Applications to Boundary Value and Expansion Problems* and it led to the award of his Ph.D. in 1907. It was an important thesis, not just for the results which it contained but also for the fact that the work had natural important extensions. Birkhoff himself developed the ideas further in the following years, as did two of his students, Rudolph Langer and Marshall Stone. For example Birkhoff and Langer published an important extension in 1923. Birkhoff's work on linear differential equations, difference equations and the generalised Riemann problem mostly all arose from the basis he laid in his thesis.

Birkhoff taught at the University of Wisconsin at Madison as an instructor from 1907 to 1909. During this period, in fact in 1908, he married Margaret Elizabeth Grafius; they had three children. After Wisconsin Madison, he went to Princeton as a preceptor in mathematics, becoming a professor there in 1911. In the following year he moved to Harvard as an assistant professor, he was promoted to full professor there in 1919, remaining at Harvard for the rest of his life. He was named Perkins Professor at Harvard in 1932, then in 1936 he became Dean of the Faculty of Arts and Science.

Because Birkhoff worked on so many different mathematical topics it is difficult to do justice to the range of his contributions in a biography of this length. However we should note that his main work was on dynamics and ergodic theory. His ergodic theorem transformed the Maxwell-Boltzmann kinetic theory of gases into a rigorous principle through the use of Lebesgue measure. Butler writes in [6]:-

Birkhoff's discovery of what has come to be known as the "ergodic theorem" in 1931 - 32 *is his most well-known contribution to dynamics. This theory, which resolved in principle one of the fundamental problems arising in the theory of gases and statistical mechanics, has been influential not only in dynamics itself but also in probability theory, group theory, and functional analysis.*

It is, of course, not only the ergodic theorem that made Birkhoff the most famous mathematician in America in his day. He had already achieved this distinction in most mathematicians eyes many years earlier when he proved Poincaré's Last Geometric Theorem, a special case of the 3-body problem, in 1913. Poincaré had stated his theorem in *Sur un théorème de géométrie* in 1912 but could only give a proof in certain special cases. Birkhoff's proof in 1913 was [1]:-

.. one of the most exciting mathematical events of the era.

The foundations of relativity and quantum mechanics were also topics which Birkhoff studied. Jointly with R E Langer, he published the monograph *Relativity and Modern Physics* in 1923. Near the end of his life he published a more speculative work, combining his ideas on philosophy and science, in *Electricity as a Fluid* in 1938. He also did important work on the four colour theorem. He developed a mathematical theory of aesthetics which he applied to art, music and poetry. Before writing *Aesthetic measure* he spent a year travelling round the world studying art, music and poetry and various countries. This is referred to in [5]:-

He has told us that the formal structure of western music, the riddle of melody, began to interest him in undergraduate days; somewhat intense consideration of the mathematical elements here involved led him to apply his theory also to aesthetic objects such as polygons, tilings, vases, and even poetry.

Among his works, some of which we have already mentioned above, are *Relativity and Modern Physics* (1923), *Dynamical Systems* (1928), *Aesthetic Measure* (1933), and *Basic Geometry* (1941).

In 1923 the American Mathematical Society made the first award of the Bôcher Memorial Prize to Birkhoff for his memoir, *Dynamical systems with two degrees of freedom* which he had published in the *Transactions of the American Mathematical Society* in 1917. He had a long association with the American Mathematical Society being Vice-President in 1919, Colloquium lecturer in 1920 when he lectured on Dynamical Systems, he edited the *Transactions of the American Mathematical Society* from 1921 to 1924 and was President from 1925 to 1926.

Perhaps this high level of involvement with the American Mathematical Society already suggest that Birkhoff worked tirelessly to advance mathematics in America. In fact Veblen described his efforts in this direct as "a sort of religious devotion". He realised, of course, that advancing mathematics in America meant having close contacts with advances in the rest of the world. His close contacts with mathematicians in Europe made him a natural person to write a report for the International Education Board on the state of mathematics in Europe. Among the European mathematicians his closest friends were Hadamard, Niels Nörlund, Levi-Civita, and Whittaker.

Of local interest in St Andrews is the fact that Birkhoff was one of the main speakers at the 1926 St Andrews Colloquium and was elected an honorary member of the Edinburgh Mathematical Society in 1927. He gave four lectures at the Gregory Tercentenary celebrations by the Edinburgh Mathematical Society in July 1938 and was awarded an honorary LL.D. by the University of St Andrews during his 1938 visit. This was, in fact, one of thirteen honorary degrees which he received.

Other honours which Birkhoff received include the Querini-Stampalia prize from the Royal Venice Institute of Science in 1917 for his paper *The restricted problem of three bodies* published in 1915, the annual prize from the American Association for the Advancement of Science in 1926, and the biennial prize from the Pontifical Academy of Sciences in Rome in 1935. He was elected to the National Academy of Sciences, the American Philosophical Society, the American Academy of Arts and Sciences, the Académie des Sciences in Paris, the

Pontifical Academy, the Circolo Matematico di Palermo, the Royal Danish Academy of Sciences and Letters, the Göttingen Academy, the Royal Institute of Bologna, the Edinburgh Mathematical Society, the London Mathematical Society, and the National Academy of Sciences of Lima, Peru. This is remarkable world-wide recognition for Birkhoff's outstanding contribution.

The article [5], written shortly before his death, describes him as:-

... by nature intensely social; through many trips to Europe, and circumnavigation of the globe, he has been extensively in contact with scholars throughout the world; he is a constant attendant and participant at meetings and congresses, and has been frequently in demand as a writer and speaker on popular themes. In recent administrative gatherings the originality and breadth of his views have been noteworthy.

There was, however, a negative side to Birkhoff's character which we should comment on. Einstein said:-

G D Birkhoff is one of the world's great anti-Semites.

In a similar vein Chandler Davis, in a personal communication, writes:-

G D Birkhoff was an early teacher of mine, and his son Garrett was my (*much appreciated*) *thesis supervisor. G D* (*but not Garrett*) *was consistently anti-Semitic, as shown in correspondence over the years; see R Phillips' article* [15] *and S Mac Lane coming to Birkhoff's defence* [11]. *He systematically kept Jews out of his department, but apparently relented late in life and favoured appointing ONE by the* 1940*s. He also helped some Jewish refugees find jobs NOT at Harvard in the* 1930*s, while acting generally to hinder their entry. Though his record is mixed and some were more implacably anti-Semitic than he was, his actions in this regard are important because of his very great influence. However, it does not seem to be true* (*as rumoured credibly at the time*) *that he opposed the appointment of Oscar Zariski to his department. As I mentioned, Garrett was not anti-Semitic at all.*

Mac Lane [11], defends Birkhoff by saying that whatever "diffuse and varied versions of anti-Semitism" Birkhoff may have had, they were undoubtedly shared by many of his contemporaries.

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