Henri Léon Lebesgue

Born: 28 June 1875 in Beauvais, Oise, Picardie, France
Died: 26 July 1941 in Paris, France

Henri Lebesgue's father was a printer. Henri began his studies at the Collège de Beauvais, then he went to Paris where he studied first at the Lycée Saint Louis and then at the Lycée Louis-le-Grand.

Lebesgue entered the École Normale Supérieure in Paris in 1894 and was awarded his teaching diploma in mathematics in 1897. For the next two years he studied in its library where he read Baire's papers on discontinuous functions and realised that much more could be achieved in this area. Later there would be considerable rivalry between Baire and Lebesgue which we refer to below. He was appointed professor at the Lycée Centrale at Nancy where he taught from 1899 to 1902. Building on the work of others, including that of the Emile Borel and Camille Jordan, Lebesgue formulated the theory of measure in 1901 and in his famous paper *Sur une généralisation de l'intégrale définie,* which appeared in the *Comptes Rendus* on 29 April 1901, he gave the definition of the Lebesgue integral that generalises the notion of the Riemann integral by extending the concept of the area below a curve to include many discontinuous functions. This generalisation of the Riemann integral revolutionised the integral calculus. Up to the end of the 19^{th} century, mathematical analysis was limited to continuous functions, based largely on the Riemann method of integration.

His contribution is one of the achievements of modern analysis which greatly expands the scope of Fourier analysis. This outstanding piece of work appears in Lebesque's doctoral dissertation, *Intégrale, longueur, aire* , presented to the Faculty of Science in Paris in 1902, and the 130 page work was published in Milan in the *Annali di Matematica* in the same year. Having graduated with his doctorate, Lebesgue obtained his first university appointment when in 1902 he became mâitre de conférences in mathematics at the Faculty of Science in Rennes. This was in keeping with the standard French tradition of a young academic first having appointments in the provinces, then later gaining recognition in being appointed to a more junior post in Paris. On 3 December 1903 he married Louise-Marguerite Vallet and they had two children. However the marriage only lasted until 1916 when they were divorced.

One honour which Lebesgue received at an early stage in his career was an invitation to give the Cours Peccot at the Collège de France. He did so in 1903 and then received an invitation to present the Cours Peccot two years later in 1905. Lebesgue first fell out with Baire in 1904, when Baire gave the Cours Peccot at the Collège de France, over who had the most right to teach such a course. Their rivalry turned into a more serious argument later in their lives. Lebesgue wrote two monographs *Leçons sur l'intégration et la recherche des fonctions primitives* (1904) and *Leçons sur les séries trigonométriques* (1906) which arose from these two lecture courses and served to make his important ideas more widely known. However, his work received a hostile reception from classical analysts, especially in France. In 1906 he was appointed to the Faculty of Science in Poitiers and in the following year he was named professor of mechanics there.

Let us attempt to indicate the way that the Lebesgue integral enabled many of the problems associated with integration to be solved. Fourier had assumed that for bounded functions term by term integration of an infinite series representing the function was possible. From this he was able to prove that if a function was representable by a trigonometric series then this series is necessarily its Fourier series. Now there is a problem here, namely that a function which is not Riemann integrable may be represented as a uniformly bounded series of Riemann integrable functions. This shows that Fourier's assumption for bounded functions does not hold.

In 1905 Lebesgue gave a deep discussion of the various conditions Lipschitz and Jordan had used in order to ensure that a function $f(x)$ is the sum of its Fourier series. What Lebesgue was able to show was that term by term integration of a uniformly bounded series of Lebesgue integrable functions was always valid. This now meant that Fourier's proof that if a function was representable by a trigonometric series then this series is necessarily its Fourier series became valid, since it could now be founded on a correct result regarding term by term integration of series. As Hawkins writes in [1]:-

In Lebesgue's work ... the generalised definition of the integral was simply the starting point of his contributions to integration theory. What made the new definition important was that Lebesgue was able to recognise in it an analytic tool capable of dealing with - and to a large extent overcoming - the numerous theoretical difficulties that had arisen in connection with Riemann's theory of integration. In fact, the problems posed by these difficulties motivated all of Lebesgue's major results.

He was appointed mâitre de conférences in mathematical analysis at the Sorbonne in 1910. During the first world war he worked for the defence of France, and at this time he fell out with Borel who was doing a similar task. Lebesgue held his post at the Sorbonne until 1918 when he was promoted to Professor of the Application of Geometry to Analysis. In 1921 he was named as Professor of Mathematics at the Collège de France, a position he held until his death in 1941. He also taught at the École Supérieure de Physique et de Chimie Industrielles de la Ville de Paris between 1927 and 1937 and at the École Normale Supérieure in Sèvres.

It is interesting that Lebesgue did not concentrate throughout his career on the field which he had himself started. This was because his work was a striking generalisation, yet Lebesgue himself was fearful of generalisations. He wrote:-

Reduced to general theories, mathematics would be a beautiful form without content. It would quickly die.

Although future developments showed his fears to be groundless, they do allow us to understand the course his own work followed.

He also made major contributions in other areas of mathematics, including topology, potential theory, the Dirichlet problem, the calculus of variations, set theory, the theory of surface area and dimension theory. By 1922 when he published *Notice sur les travaux scientifique de M Henri Lebesgue* he had written nearly 90 books and papers. This ninety-two page work also provides an analysis of the contents of Lebesgue's papers. After 1922 he remained active, but his contributions were directed towards pedagogical issues, historical work, and elementary geometry.

Lebesgue was honoured with election to many academies. He was elected to the Academy of Sciences on 29 May 1922, to the Royal Society, the Royal Academy of Science and Letters of Belgium (6 June 1931), the Academy of Bologna, the Accademia dei Lincei, the Royal Danish Academy of Sciences, the Romanian Academy, and the Krakow Academy of Science and Letters. He was also awarded honorary doctorates from many universities. He also received a number of prizes including the Prix Houllevigue (1912), the Prix Poncelet (1914), the Prix Saintour (1917) and the Prix Petit d'Ormoy (1919).

Article by: *J J O'Connor* and *E F Robertson*