Joseph Liouville

Born: 24 March 1809 in Saint-Omer, France
Died: 8 Sept 1882 in Paris, France

Joseph Liouville's father was an army captain in Napoleon's army so Joseph had to spend the first few years of his life with his uncle. His father was certainly fortunate to survive the wars and after Napoleon was defeated he retired to live with his family. The family then settled in Toul where Joseph attended school. From Toul he went to the Collège St Louis in Paris where he studied mathematics at the highest levels. After reading articles in Gergonne's Journal he proved some geometrical results which he wrote up as papers although they were never published.

Liouville entered the École Polytechnique in 1825 and attended Ampère's *Cours d'analyse et de mécanique* in session 1825-26. He also attended courses by Arago at the École Polytechnique as well as a second course by Ampère at the Collège de France. Although Liouville does not seem to have attended any of Cauchy's courses, it is clear that Cauchy must have had a strong influence on him. Liouville graduated in 1827 with de Prony and Poisson among his examiners.

After graduating from the École Polytechnique Liouville entered the École des Ponts et Chaussées. However his health suffered when he had to undertake engineering projects and he spent some time at his home in Toul recovering. By now Liouville was set on an academic career and he found it impossible to study away from Paris. After a number of periods of leave, one of which allowed him to marry and have a few days honeymoon, it became clear to him that he must resign from the École des Ponts et Chaussées. This he did in October of 1830 but even at this stage he had written a number of papers which he had submitted to the Paris Academy on electrodynamics, partial differential equations and the theory of heat.

In 1831 Liouville was appointed to his first academic post, as assistant to Claude Mathieu who had been appointed to Ampère's chair at the École Polytechnique. He was also appointed to a number of private schools and to the École Centrale. It is remarkable that during this period of his life Liouville taught between 35 and 40 hours a week at the different institutions. Perhaps with a schedule this heavy it is not surprising that some courses would not go particularly well and it appears that he lectured at too high a level for some of the less able students.

In 1836 Liouville founded a mathematics journal *Journal de Mathématiques Pures et Appliquées*. This journal, sometimes known as *Journal de Liouville*, did much for mathematics in France throughout the 19th century. Liouville had already gained an international reputation with papers published in Crelle's Journal but at the same time the quality of Crelle's Journal made him aware of deficiencies in the avenues for mathematical publications which there were in France. Certainly he was unhappy with the style of the Paris Journals for he wrote in 1836:-

.. a peculiar spirit of emigration has seized some critics and we have seen them heap abuse on one after the other of the men who in various fields of science have honoured France with great dignity. ... this sharp and peremptory style ... will never be mine, for it dishonours both the character and talent of those who adopt it.

Liouville became favourite to fill the chair at the École Polytechnique which fell vacant when Navier died in 1836. However, after a close competition, Duhamel was appointed. In 1837 Liouville was appointed to lecture at the Collège de France as a substitute for Biot. In 1838 Liouville was appointed Professor of Analysis and Mechanics at the École Polytechnique.

The following year he was elected to the astronomy section of the Académie des Sciences but this was only after strong opposition from Libri. In fact the quarrel between Liouville and Libri intensified after his election to the Académie. In 1840, after a vacancy resulting from the death of Poisson, Liouville was elected to the Bureau des Longitudes.

In many ways 1840 was a turning point in Liouville's career. As Lützen writes in [4]:-

Before 1840*, Liouville had pursued some clear paths to secure his own career; during the following twenty years, the promotion and development of other mathematicians' ideas became the central issue.*

Life for Liouville developed into a year with two distinct parts. During the long summer period, spent at Toul, he undertook research, wrote papers and carried out editing duties. From November to July he lived in Paris and carried out his teaching and administrative duties.

Not everything went Liouville's way however. When Lacroix died in 1843, Liouville applied for his chair at the Collège de France where he lectured only as a substitute for Biot. However after a close election Libri was appointed. Liouville immediately resigned from the Collège de France, writing in his resignation letter:-

I am profoundly humiliated as a person and as a geometer by the events that took place yesterday at the Collège de France. From this moment it is impossible for me to lecture at this institution.

Another aspect of Liouville's life was his involvement in politics. One of his friends, and mathematical colleagues, was Arago who entered the Chamber of Deputies in 1831 and became leader of the Republican Party. Other mathematical colleagues had also become involved with the political events of the time, for example Catalan, whose political views were similar to the republican views of Liouville, had damaged his mathematical career. Liouville certainly never let his political views hold him back as he advanced his mathematical career, unlike Cauchy who had refused to swear the oaths of allegiance to the King that Liouville and even Arago had been prepared to do.

Encouraged by Arago, Liouville stood for election to the Constituting Assembly in 1848. In his recommendation of Liouville as a candidate Arago wrote:-

... Mr Liouville is one of my best friends. He is a very eminent man, a patriot, an experienced republican. God grant that the National Assembly will contain many members of this calibre.

Elected on 23 April 1848, Liouville took his seat among the moderate republican majority. However there was unrest in Paris as workers felt that their revolution had been taken over by the bourgeoisie. When Arago tried to address the crowds a heckler shouted the telling comment:-

Mr Arago, you have never been hungry.

Liouville continued his political career by being renominated for the Assembly elections in 1849 but the tide had turned against the moderate republicans and he was not elected.

The election defeat proved another turning point in Liouville's life. As Lützen writes in [4]:-

The political defeat changed Liouville's personality. In earlier letters, he was often depressed because of illness, and could vent his anger towards his enemies such as Libri, but he always fought for what he believed was right. After the election in 1849*, he resigned and became bitter, even towards his old friends. When he sat down at his desk, he did not only work, ... he also pondered his ill fate. ... his mathematical notes were interrupted with quotes from poets and philosophers...*

Libri escaped from France during the 1848 revolution, not for political reasons, but to avoid a prison sentence for stealing precious books and manuscripts. His chair at the Collège de France was declared vacant in 1850 and Cauchy and Liouville competed for the post. In a close contest Liouville triumphed and began his lectures at the Collège de France in 1851.

Although Liouville's mathematical output had been greatly reduced while he was involved with politics, it picked up again in the 1850s despite health problems. In fact 1856 and 1857 were two of Liouville's most productive years. However after being appointed to the chair of mechanics at the Faculté des Sciences in 1857 his teaching load began to take its toll on him. Not only did he have a high teaching load but Liouville was a perfectionist which meant that when he felt that he could not devote all the time necessary to give the best possible lectures he began to suffer. He continued to publish but mostly results he had discovered during his highly productive year of 1856 and these without the proofs which now he could not find time to polish.

Another blow to Liouville was the death of Dirichlet in 1859. This was a serious blow to him mathematically for, as well as losing a close friend, he lost his main mathematical correspondent.

Liouville's mathematical work was extremely wide ranging, from mathematical physics to astronomy to pure mathematics. One of the first topics he studied, which developed from his early work on electromagnetism, was a new topic, now called the fractional calculus. He defined differential operators of arbitrary order D^t. Usually t is an integer but in this theory developed by Liouville in papers between 1832 and 1837, t could be a rational, an irrational or most generally of all a complex number.

Liouville investigated criteria for integrals of algebraic functions to be algebraic during the period 1832-33. Having established this in four papers, Liouville went on to investigate the general problem of integration of algebraic functions in finite terms. His work at first was independent of that of Abel, but later he learnt of Abel's work and included several ideas into his own work.

Another important area which Liouville is remembered for today is that of transcendental numbers. Liouville's interest in this stemmed from reading a correspondence between Goldbach and Daniel Bernoulli. Liouville certainly aimed to prove that e is transcendental but he did not succeed. However his contributions were great and led him to prove the existence of a transcendental number in 1844 when he constructed an infinite class of such numbers using continued fractions. In 1851 he published results on transcendental numbers removing the dependence on continued fractions. In particular he gave an example of a transcendental number, the number now named the Liouvillian number

0.11000100000000000000010000...

where there is a 1 in place n! and 0 elsewhere.

His work on boundary value problems on differential equations is remembered because of what is called today Sturm-Liouville theory which is used in solving integral equations. This theory, which has major importance in mathematical physics, was developed between 1829 and 1837. Sturm and Liouville examined general linear second order differential equations and examined properties of their eigenvalues, the behaviour of the eigenfunctions and the series expansion of arbitrary functions in terms of these eigenfunctions.

Liouville contributed to differential geometry studying conformal transformations. He proved a major theorem concerning the measure preserving property of Hamiltonian dynamics. The result is of fundamental importance in statistical mechanics and measure theory.

In 1842 Liouville began to read Galois's unpublished papers. In September of 1843 he announced to the Paris Academy that he had found deep results in Galois's work and promised to publish Galois's papers together with his own commentary. Liouville was therefore a major influence in bringing Galois's work to general notice when he published this work in 1846 in his Journal. However he had waited three years before publishing the

papers and, rather strangely, he never published his commentary although he certainly wrote a commentary which filled in the gaps in Galois's proofs. Liouville also lectured on Galois's work and Serret, possibly together with Bertrand and Hermite, attended the course.

In number theory Liouville wrote around 200 papers, working on quadratic reciprocity and many other topics. He wrote over 400 papers in total.

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