# George Pólya

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**Born: 13 Dec 1887 in Budapest, Hungary**
**Died: 7 Sept 1985 in Palo Alto, California, USA**

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**George Pólya**'s parents were Anna Deutsch and Jakab Pólya who were both Jewish. Anna was from a family who had lived for many generations in Buda, and she had been nineteen years old in 1872 when the towns of Buda, Obuda, and Pest had administratively merged to become the city of Budapest. Perhaps we should say a little about George Pólya's names, for the situation is not quite as it appears. In fact, although Jakab Pólya had the name "Pólya" when his son György (or George as he was later known) was born, he had only called himself Pólya for the five preceding years. Before that his name had been Jakab Pollák but, in order to understand why Jakab Pollák changed his name to Pólya, we need to look at both his career and at a little Hungarian history.

Jakab was trained as a lawyer, ran his own law firm which failed, and then worked for the international insurance company Assicurazioni Generali of Trieste. However what he really wanted was a university post in which he could conduct research into the subjects which really interested him, namely economics and statistics. After 1867 Hungary had gained full internal independence within the Austro-Hungarian Monarchy and the political philosophy of the country was to move towards a Hungarian state that was both Magyar in spirit, and in its institutions. What better way for Jakab Pollák to improve his chances of a university post than to change his name from a Jewish sounding one to one which sounded really Hungarian. He did just that in 1882 and whether it contributed to his success in getting an appointment as a Privatdozent at the University of Budapest, one cannot say but he received such a post shortly before he died in his early fifties when George was ten years old.

In fact although George's parents were Jewish, he was baptized into the Roman Catholic Church shortly after his birth. How did this come about? Well Jakab, Anna, and their three children at the time, converted from the Jewish faith to the Roman Catholic faith in 1886, the year before George's birth.

When Jakab Pólya died in 1897 he left a wife, Anna aged 44 at the time, and five children. George had an older brother Jenö, who was 21 years old and studying medicine when his father died, two older sisters Ilona (10 years older than George) and Flóra (8 years older than George) who went to work for the insurance company Assicurazioni Generali to help support the family, and a younger brother Lásló (4 years younger than George). It is worth pointing out that Jenö, who loved mathematics and always regretted not having pursued that subject, is perhaps as well known to medical people as George is to mathematicians. However, it was Lásló who was considered the brightest of the children, but sadly he was killed in World War I before making a name for himself. Perhaps given how much effort his father had put in trying to enter the academic profession, it is slightly surprising that George's mother should press him to follow his father's profession of law but this is exactly what she did.

George attended elementary school in Budapest and received his certificate in 1894 which recorded (see for example [2]):-

*... diligence and good behaviour*.

Following this he entered the Dániel Berzsenyi Gymnasium studying the classical languages of Greek and Latin as well as the modern language of German and of course Hungarian. At school Pólya's favourite subjects were biology and literature and in this latter subject he received "outstanding" grades as he did in geography and other subjects. It is rather unusual that someone who went on to spend their life being so fascinated by so many different branches of mathematics should not have fallen in love with the subject at school but in Pólya's case this is exactly what happened. He did not score particularly high marks in mathematics at the Gymnasium, his work in geometry being graded as merely "satisfactory". He did score rather better in arithmetic, however. The reason for his lack of success in mathematics may well have been due to poor teaching, and he would later describe two of his three mathematics teachers at the gymnasium as "despicable teachers".

Pólya enrolled at the University of Budapest in 1905 supported financially by his brother Jenö who was by now a surgeon. He began to study law but found it so boring that he gave up that topic after one semester. He then studied his favourite school subjects of languages and literature for two years, gaining his certificate which allowed him to teach Latin and Hungarian in a gymnasium. It was a qualification of which he was proud but never put it to use. He then became very interested in philosophy but his professor, Bernát Alexander, advised him to take physics and mathematics courses to help him understand this subject, so eventually he was made to study mathematics. He made the witty remark, which should not be taken seriously [4]:-

*I thought I am not good enough for physics and I am too good for philosophy. Mathematics is in between.*

Now at the University of Budapest Pólya was taught physics by Eötvös and mathematics by Fejér. Pólya said [4]:-

*I was greatly influenced by Fejér, as were all Hungarian mathematicians of my generation, and, in fact, once or twice in small matters I collaborated with Fejér. In one or two papers of his I have remarks and he made remarks in one or two papers of mine, but it was not really a deep influence.*

The academic year 1910-11 Pólya spent studying at the University of Vienna where he earned money by teaching the son of an important local dignitary (his pupil, apparently, lacking any talent whatsoever). In Vienna he attended mathematics lectures by Wirtinger and Mertens but continued to have a strong interest in physics attending lectures in relativity, optics and other topics. In the following year he returned to Budapest where he was awarded a doctorate in mathematics having studied, essentially without supervision, a problem in the theory of geometric probability. He then spent much of 1912 and 1913 at Göttingen where he mixed with a whole host of leading mathematicians such as Klein, Carathéodory, Hilbert, Runge, Edmund Landau, Weyl, Hecke, Courant and Toeplitz.

In fact Pólya left Göttingen in rather unfortunate circumstances. He explained the incident in a letter to Bieberbach in 1921 (see for example [2]):-

*On Christmas* 1913 *I travelled by train from Zürich to Frankfurt and at that time I had a verbal exchange - about my basket that had fallen down - with a young man who sat across from me in the train compartment. I was in an overexcited state of mind and I provoked him. When he did not respond to my provocation, I boxed his ear. Later on it turned out that the young man was the son of a certain Geheimrat; he was a student, of all things, in Göttingen. After some misunderstandings I was told to leave by the Senate of the University.*

He received an offer of an appointment at Frankfurt but, before taking up this appointment, he went to Paris for a short visit early in 1914, meeting Emile Picard and Hadamard but not enjoying his visit a great deal mainly due to dreadful accommodation. From the wide range of mathematical stars that Pólya had met the mathematician who was the greatest influence on him was Hurwitz. Therefore when Pólya learnt during his stay in Paris that Hurwitz had arranged an appointment as Privatdozent for him at Eidgenössische Technische Hochschule Zürich, where Hurwitz himself held the chair of mathematics, Pólya decided to accept [4]:-

*I was... deeply influenced by Hurwitz. In fact I went to Zurich in order to be near Hurwitz and we were in close touch for about six years, from my arrival in Zürich in* 1914 *to his passing in ...* 1919*. And we have one joint paper, but that is not the whole extent. I was very much impressed by him and edited his works. I was also impressed by his manuscripts.*

In Zürich, in addition to Hurwitz, Pólya had Geiser, Bernays, Zermelo and Weyl as colleagues. Of course his arrival in Zürich was in the year that World War I started, but at first this caused Pólya no real problems since a soccer injury he had received as a student meant that he was not deemed medically fit for service in the Hungarian army. This was fortunate for him since, by this time, he held firm pacifist views. Life became more difficult as the war progressed, however, since the Hungarian army, becoming more desperate for soldiers as the war progressed, required Pólya to return to Hungary, to join the army, and to fight for his country; he refused. This did have the consequence that it would be many years after the war ended before Pólya was able to return to Hungary without fear of arrest for failing to undertake war service. He took Swiss citizenship, although this did not protect him from the Hungarian authorities, and in 1918 he married a Swiss girl, Stella Vera Weber, who was a daughter of the professor of physics at the University of Neuchâtel. In fact, although it is difficult to see why he waited so long, Pólya did not return to Hungary until 1967, 54 years after his last visit to his native land.

Pólya first met Szegö on Budapest in around 1913 when he returned there between his various studies abroad. Szegö at this time was a student at Budapest and Pólya discussed a conjecture he had made on Fourier coefficients with Szegö. In fact Szegö went on to prove Pólya's conjecture and this became his first publication. When several years later Pólya decided to write a problem book on analysis he knew that it was not a task he could accomplish without help, so he turned to Szegö and over a number of years the two assembled a wonderful collection of problems. In [4] Pólya explained why he approached putting across mathematical ideas in a different way to that previously used:-

*I came very late to mathematics. ... as I came to mathematics and learned something of it, I thought: Well it is so, I see, the proof seems to be conclusive, but how can people find such results? My difficulty in understanding mathematics: How was it discovered?*

What was the great novelty which made Pólya and Szegö's book of analysis problems so different? It was Pólya's idea to classify the problems not by their subject, but rather by their method of solution. Pólya and Szegö approached the publisher Springer in 1923 with their idea for a two volume work and in 1925 *Aufgaben und Lehrsätze aus der Analysis* appeared. This work was [5]:-

*... a mathematical masterpiece that assured their reputations.*

Pólya had been promoted to extraordinary professor at ETH in Zürich in 1920. He received a Rockefeller Fellowship in 1924 to enable him to study with Hardy in England. He spent 1924 partly in Oxford, partly in Cambridge, working with Hardy and Littlewood and they began a collaboration on the book *Inequalities* was published in 1934. While the book was being worked on, Pólya continued a remarkable series of publications, with a total of 31 papers appearing during the three years 1926-28. Given the range, depth and number of these publications it is not surprising that he was promoted to Ordinary Professor at ETH in 1928.

In reviewing [3], Duren gave this summary of Pólya's mathematical achievements:-

*Pólya was arguably the most influential mathematician of the* 20*th century. His basic research contributions span complex analysis, mathematical physics, probability theory, geometry, and combinatorics. He was a teacher par excellence who maintained a strong interest in pedagogical matters throughout his long career*.

While in Zürich his output of mathematics was very large and wide ranging. For example, in 1918 he published papers on series, number theory, combinatorics and voting systems. The following year, in addition to papers on these topics, he published on astronomy and probability. While he was doing this wide range of work he was also proving some of his deepest results in the study of integral functions.

In 1933 Pólya was awarded a second Rockefeller Fellowship, this time to allow him to visit Princeton. While he was in the United States Blichfeldt invited him to visit Stanford which he did, and greatly enjoyed being there. He returned to Zürich but in 1940 the political situation in Europe forced Pólya to move to the United States where, after working at Brown University for two years and Smith College for a short while, he took up an appointment at Stanford. Before going to the United States Pólya had a draft of a book *How to solve it* written in German. He had to try four publishers before finding one to publish the English version in the United States but it sold over one million copies over the years and has been translated in 17 languages. Schoenfeld described its importance in [24]:-

*For mathematics education and the world of problem solving it marked a line of demarcation between two eras, problem solving before and after Pólya.*

Pólya explained in *How to solve it* that to solve problems required the study of heuristic:-

*The aim of heuristic is to study the methods and rules of discovery and invention .... Heuristic, as an adjective, means 'serving to discover'. ... its purpose is to discover the solution of the present problem. ... What is good education? Systematically giving opportunity to the student to discover things by himself.*

He also gave the wise advice:-

*If you can't solve a problem, then there is an easier problem you can't solve: find it.*

Pólya published further books on the art of solving mathematical problems. For example *Mathematics and plausible reasoning* (1954), and *Mathematical discovery* which was published in two volumes (1962, 1965).

While we are looking at Pólya's contributions to teaching, and many people consider this to be his greatest contribution to mathematics, let us give some further quotes from Pólya on this topic. First a quote from a lecture on teaching mathematics in primary schools:-

*Mathematics in the primary schools has a good and narrow aim and that is pretty clear in the primary schools. ... However, we have a higher aim. We wish to develop all the resources of the growing child. And the part that mathematics plays is mostly about thinking. Mathematics is a good school of thinking. But what is thinking? The thinking that you can learn in mathematics is, for instance, to handle abstractions. Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem. ... But I think there is one point which is even more important. Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.*

Next we give a quote from Pólya regarding teaching in general:-

*Teaching is not a science; it is an art. If teaching were a science there would be a best way of teaching and everyone would have to teach like that. Since teaching is not a science, there is great latitude and much possibility for personal differences. ... let me tell you what my idea of teaching is. Perhaps the first point, which is widely accepted, is that teaching must be active, or rather active learning. ... the main point in mathematics teaching is to develop the tactics of problem solving.*

Let us briefly discuss some of the research which Pólya carried out in many different areas of mathematics. It was so wide ranging and so plentiful that there is no way that we can do more than mention a few aspects. In probability Pólya looked at the Fourier transform of a probability measure, showing in 1923 that it was a characteristic function. He wrote on the normal distribution and coined the term "central limit theorem" in 1920 which is now standard usage. In 1921 he proved his famous theorem on random walks on an integer lattice. He considered a $d$-dimensional array of lattice points where a point moves to any of its neighbours with equal probability. He asked whether given an arbitrary point $A$ in the lattice, a point executing a random walk starting from the origin would reach $A$ with probability 1. Pólya's surprising answer was that it would for $d = 1$ and for $d = 2$, but it would not for $d \geq 3$. In later work he looked at two points executing independent random walks and also at random walks satisfying the condition that the moving point never passed through the same lattice point twice.

Geometric symmetry and the enumeration of symmetry classes of objects was a major area of interest for Pólya over many years. He added to the understanding of the 17 plane crystallographic groups in 1924 by illustrating each with tilings of the plane. This paper inspired Escher to produce his famous work on periodic drawings. Pólya's work using generating functions and permutation groups to enumerate isomers in organic chemistry was of fundamental importance.

His main contribution to combinatorics is his enumeration theorem, published in 1937. Read, in [18], describes this as:-

*... a remarkable theorem in a remarkable paper, and a landmark in the history of combinatorial analysis.*

The theorem solves the problem of how many configurations with certain properties exist. It has applications such as the enumeration of chemical compounds and the enumeration of rooted trees in graph theory. In fact a whole new area of graph theory called enumerative graph theory grew up based on Pólya's ideas.

Pólya's interest in complex analysis led him to investigate singularities of power series, gap theorems, power series with integral coefficients and those taking integral values at the positive integers, the Pólya representation for entire functions of exponential type, and the location of zeros. He also worked on conformal mappings and potential theory, and he was led to study boundary value problems for partial differential equations and the theory of various functionals connected with them. His methods applied particularly to isoperimetric problems in domains with a high degree of symmetry. Together with Szegö, he wrote the now classic text *Isoperimetric inequalities in mathematical physics* in 1951. Schiffer writes in [23]:-

*The whole work displays the taste of the authors for the concrete and explicit result, for elegance and ingenious methods.*

In 1953 Pólya retired from Stanford, but continued with an exceedingly active mathematical life particularly concerning himself with mathematical education. He continued his association with Stanford as Professor Emeritus and, on 13 December 1977, a dinner was given there to mark his $90^{th}$ birthday at which many friends and colleagues gave glowing tributes. His teaching career, however, was still not over and in 1978 he taught a course on combinatorics in the Computer Science Department at Stanford.

He received many honours for his outstanding contributions and we only mention a few here. He was elected an honorary member of the Hungarian Academy, the London Mathematical Society, the Mathematical Association of Great Britain, and the Swiss Mathematical Society. He was elected to the National Academy of Sciences of the United States, the American Academy of Arts and Sciences, the Académie Internationale de Philosophie des Sciences de Bruxelles, and the California Mathematics Council. he was a corresponding member of the Académie des Sciences in Paris.

Let us end this article with Frank Harary's tribute to Pólya [11]:-

*With no hesitation, George Pólya is my personal hero as a mathematician. ...* [*he*] *is not only a distinguished gentleman but a most kind and gentle man: his ebullient enthusiasm, the twinkle in his eye, his tremendous curiosity, his generosity with his time, his spry energetic walk, his warm genuine friendliness, his welcoming visitors into his home and showing them his pictures of great mathematicians he has known - these are all components of his happy personality. As a mathematician, his depth, speed, brilliance, versatility, power and universality are all inspiring. Would that there were a way of teaching and learning these traits.*

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