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Test	#2

## **AMATYC Student Mathematics League**

February-March 2003

1. If  $log_b 12 = 2.5$ , which of the following is closest to b?

A. 2.5 B. 2.7 C. 3.0 D. 3.2 E. 3.4

2. If the ordered pair (200,3) is a solution to the equation ax + by = c, which of the following must be a solution to the equation bx - ay = c?

A. (3, -200) B. (-3, 200) C. (3, 200) D. (200, -3) E. (-200, 3)

3. If  $i = \sqrt{-1}$ , what is  $i^{-321}$ ?

A. 0 B. 1 C. -1 D. i E. -i

4. In triangle SML,  $m \angle S = 30^{\circ}$  and  $m \angle M = 40^{\circ}$ . If P is chosen on side SM so that segment LP bisects  $\angle$ SLM, what is  $m \angle$ SPL?

A. 70° B. 85° C. 90° D. 95° E. 110°

5. Square MATH is placed with vertices A and T on a circle of diameter 20 so that the midpoint of side MH is the circle's center. The area of the square is

A. 40 B. 60 C. 80 D. 90 E. 100

6. Using any number of 7 g and 11 g weights and a balance scale (2 pans joined by an arm), what is the least number of weights needed to balance a 1 g object?

A. 5 B. 6 C. 7 D. 8 E. 9

7. Let P(x + 2) = 3x + 1. What value of x satisfies the equation P(x + 3) = 13?

A. 3 B. 5 C. 40 D. 16 E. 15

8. How many real solutions does the equation  $\sqrt{x^3 - x^2 - x + 2} = x$  have?

A. 1 B. 2 C. 3 D. 4 E. 6

9. The consecutive integers from 5 to 13 are placed in a 3x3 grid, one per square, so that each row, column, and long diagonal sums to the same value. If 6 is in the lower left square and 5 is in the middle right square as shown, what number is in the bottom middle square (the asterisk)?

5 6 \*

- A. 7 B. 8 C. 9 D. 10 E. 11
- 10. Reverse my age, divide by three, add twenty, and my age you'll see. How old am I? Write your numerical answer in the corresponding space on the answer sheet.
- 11. Let  $a_n = \sum_{k=1}^{n} k(-1)^k$ . What is  $a_{2003}$ ?

A. 1001 B. -1001 C. 1002 D. -1002 E. 1003

12. A parabola with vertical axis is shifted 1 unit right and 1 unit up so that it intersects the original parabola at the point (1,3). What is the y-intercept of the original parabola?

A. 2 B. -2 C. 3 D. -3 E. 4

13. A circle of radius 4 lies in the interior of a 60° angle so that it is tangent to both sides of the angle. If a smaller circle is tangent to both sides of the angle and to the larger circle, what is its radius?

A. 1 B.  $\frac{3}{2}$  C. 2 D.  $\frac{4}{3}$  E.  $\frac{5}{3}$ 

14. If f(x) = ax + b (a and b integers) and f(f(x)) = 4x + 5, then f(2) = 4x + 5

A. -9 B. -7 C. 0 D. 1 E. 3

15. Let R be the region in the plane consisting of all points (x,y) such that  $|x-y|+|x|+|y| \le 50$ . The area of R is

A. 625 B. 1250 C. 1875 D. 2500 E. infinite

16. The side lengths, semiperimeter (half the perimeter), and the area of a triangle form consecutive terms of an arithmetic sequence. What is the perimeter?

A. 5/13 B. 19/2 C. 14 D. 49/3 E. 23

17. A positive integer is called "strange" if its prime factorization consists of exactly two factors, of which exactly one is less than 10. For example, 26 = (2)(13) is strange, but 35 = (3)(7), 66 = 2(3)(11), and 143 = (11)(13) are not. Let A, A+1, and A+2 be the three smallest consecutive positive integers such that all three are strange. What is the larger prime factor of A?

A. 13 B. 17 C. 19 D. 29 E. 31

18. Three distinct integers whose sum is 42 form a geometric sequence with common ratio an integer. How many such sequences are there?

A. 1 B. 3 C. 5 D. 6 E. 8

19. Suppose x and y are acute angles with  $\cos x = \tan y$  and  $\cos y = \tan x$ . What is the value of  $(\sin x)^2 + \sin x$ ?

A.  $\frac{1}{2}$  B.  $\frac{-1+\sqrt{5}}{2}$  C.  $\frac{1+\sqrt{5}}{2}$  D.  $\frac{3}{4}$  E. 1

A piece of a 10-inch diameter pie is made by cutting 2 inches straight from the edge (not necessarily toward the center), then making a right angle and cutting straight back to the edge. If the second cut is 6 inches long, what is the area in square inches of the piece (to the nearest tenth)?

A. 10.1 B. 10.3 C. 10.5 D. 10.7 E. 10.9

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	Student's Responses	Local Corrector	
1	В		
2	Α		
3	E		
4	D		
5	С		
6	A		
7	A		
8	В		_
9	E		
10	48		
11	D		
12	Α		
13	D		
14	A		
15	С		
16	С		
17	В		
18	С		
19	E		
20	E		

COLLEGE:
ROUND: 1 2
# correct =
# incorrect =
# blank =
= # correct × 2  = # incorrect × 1/2  = score