Tes	t #2			AM	AMATYC Student Mathematics League							February/March 2006	
1. If $f(x) = \cos \pi x$ and $g(x) = 2x$, find $f(g(1)) - g(f(1))$.													
A.	-3	В.	-1	C.	0	D.	1	E.	3				
2.	How	⁷ many	, differ	ent fou	r-digit	numbe	rs can	be forn	ned by	arrang	ging the	e digits 2, 0, 0, and 6?	
A.	6	B.	8	C.	10	D.	12	E.	24			C	
3.	If ABCD, DCEF, and FEGH are squares with A, B, C, D, E, F, G, and H all distinct points, find $m \angle GAH + m \angle GDH + m \angle GFH$ to the nearest tenth of a degree.												
А.	80°	В.	87.5	C	C.	90°	D.	92.5	C	E.	100°		
4.	I sold a horse for \$200, losing 20%. I bought another horse and sold it for a 25% profit. If I broke even on the two transactions together, what was the total cost of the two horses?												
А.	\$432	В	. \$45	0	C.	\$500		D.	\$540		E. \$	562.50	
5.	Let A(m,n) be the set of n consecutive positive integers whose least element is m. What is the greatest integer in A(17,49) \cap A(49,17)?												
A.	33	В.	49	C.	65	D.	66	E.	67				
6.	Let a	Let $a, b > 0, M = \sum_{n=1}^{k} \ln(an) - \sum_{n=1}^{k} \ln(bn), N = e^{M}$, and $P = \sqrt[k]{N}$. Then P equals $\frac{a}{b}$ B. $a-b$ C. $\sqrt[k]{k(a-b)}$ D. $\sqrt[k]{\frac{ka}{b}}$ E. $e^{a/bk}$											
A.	$\frac{a}{b}$	В.	a – ł)	C.	$\sqrt[k]{k(a)}$	<i>-b</i>)	D.	$\sqrt[k]{\frac{ka}{b}}$		E.	$e^{a/bk}$	
7.	Whie	ch of th	ne follo	wing i	mply tl	hat the 1	real nu	umber >	x must	be rati	onal?		
	II. x	Which of the following imply that the real number x must be rational? I. x^5 , x^7 are both rational II. x^6 , x^8 are both rational III. x^5 , x^8 are both rational											
А. Е.			B. ese com			C.	II, II	I only	D.	III o	nly		
8.	A positive integer less than 1000 is chosen at random. What is the probability it is a multiple of 3, but a multiple of neither 2 nor 9 ?												
A.	$\frac{1}{10}$	B.	$\frac{1}{9}$	C.	$\frac{1}{8}$	D.	$\frac{2}{9}$	E.	$\frac{1}{3}$				
9.	Let r	Let r and s be the solutions to the equation $x^2 + 3x + c = 0$. If $r^2 + s^2 = 33$, find the value of c.											
A.	-21	В.	-12	C.	1	D.	12	E.	21				
10.	Joe must determine the greatest whole number of feet from which a ceramic ball can be dropped without breaking. He has two identical ceramic balls which he can drop from any whole numbered height he wants. If he must determine this height with no more than 12 drops, what is the greatest height for which he can determine this with certainty?												
А.	20-25	5ft B	. 26-4	0 ft C	2. 41-5	0 ft D.	51-7	5 ft E	. more	e than	75 ft		

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- 11. In convex pentagon AMTYC, $\overrightarrow{CY} \perp \overrightarrow{YT}$, $\overrightarrow{MT} \perp \overrightarrow{YT}$, $\overrightarrow{CY} = \overrightarrow{YT} = 63$, MT = 79, AM = 39, and AC = 52. Find the area of the pentagon.
- A. 5487 B. 5500 C. 5525 D. 5600 E. 5624
- 12. The *midrange* of a set of numbers is the average of the greatest and least values in the set. For a set of six increasing nonnegative integers, the mean, the median, and the midrange are all 5. How many such sets are there?
- A. 10 B. 12 C. 20 D. 24 E. 30
- 13. The sum of the absolute values of all solutions of the equation $|x^3 + 4x^2 6x 22| = x^2 + 2x + 2$ can be written in the form $a + b\sqrt{c}$, c a prime. Find a + b + c.
- A. 12 B. 14 C. 16 D. 17 E. 18
- 14. Find the number of three-digit numbers containing no even digits which are divisible by 9.
- A. 8 B. 9 C. 10 D. 11 E. 12
- 15. If α is the acute angle formed by the lines with equations y = 2x 5 and y = 1 3x, find $\tan \alpha$. A. $\frac{1}{\sqrt{3}}$ B. $\frac{1}{2}$ C. 1 D. 2 E. $\sqrt{3}$
- 16. Find the number of points of intersection of the unit circle and the graph of the equation $y^2 xy |x|y + x|x| = 0$.
- A. 0 B. 1 C. 2 D. 3 E. 4
- 17. Suppose that for a function y = f(x), f(x) > x for all x. Let A be the point with x-coordinate *a* on the function y = f(x) and B be the point on the graph of the line y = x for which \overline{AB} is perpendicular to the line. Find an expression for the distance from A to B.

A.
$$(f(a) - a)\sqrt{2}$$
 B. $a\frac{\sqrt{2}}{2}$ C. $(f(a) - a)\frac{\sqrt{2}}{2}$ D. $f(a) - a$ E. $f(a)\sqrt{2}$

- 18. In the quadrilateral PQRS, PQ = 1, QR = RS = $\sqrt{2}$, PS = $\sqrt{3}$, and QS = 2. If T is the point of intersection of the diagonals, find the measure in degrees of angle RTS.
- A. 45 B. 55 C. 60 D. 75 E. 105
- 19. Call a composite number *circumfactorable* if all of its positive integer factors greater than 1 can be placed around a circle so that any two adjacent factors have a common factor greater than 1. How many composite numbers less than 200 are not circumfactorable?
- A. 50 B. 52 C. 54 D. 56 E. 58
- 20. A circle contains 2006 points chosen so that the arcs between any two adjacent points are equal. Three of these points are chosen at random. Let the probability that the triangle formed is right be *R*, and the probability that the triangle formed is isosceles be *I*. Find |R I|.
- A. 0 B. 1/5 C. 1/4 D. 1/3 E. 1/2

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