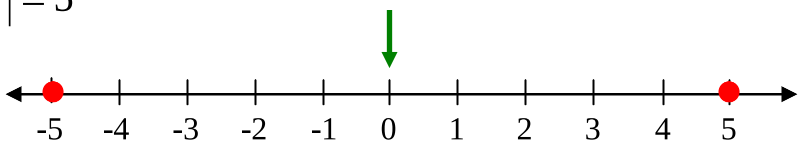
18.3 Absolute-Value Equations and Inequalities

❖ **<u>Absolute Value</u>:** the distance from zero on a number line.

Ex. $|x| = 5$



The variable can have a value of either 5 or -5 because both of these numbers are 5 units from zero on the number line.

Absolute Value Property

If $|x| = a$, where x is a variable or an expression and $a \geq 0$, then $x = a$ or $x = -a$.

Ex. Solve. Write the solution in set notation.

(a) $|x| = 9$ (b) $|p| = 0$ (c) $|x| = -3$

In general, the following is true:

If a is ***positive***, the equation $|x| = a$ has **two solutions**, a and $-a$.

If a is **0**, the equation $|x| = a$ has **one solution**, 0.

If a is ***negative***, the equation $|x| = a$ has **no solution**.

Ex. Solve. Write the solution in set notation.

(a) $|x+2|=3$.

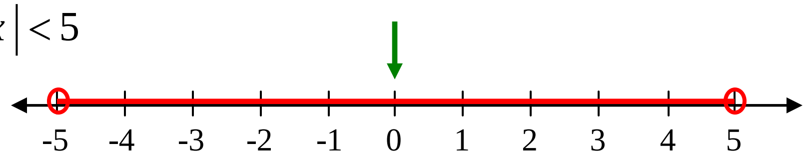
(b) $|3x|=-8$

(c) $|5x|-3=37$

(d) $3-2|x|=-7$

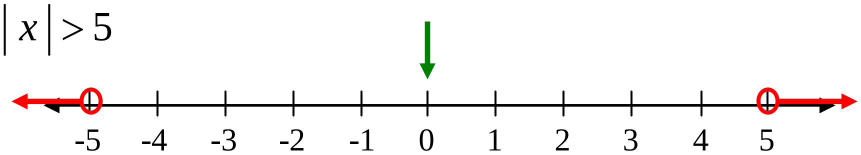
❖ ***Absolute Value Inequalities***

Ex. $|x|<5$



The solution set of $|x|<5$ contains all numbers whose distance from 0 is <u>less than</u> 5 units on the number line.

Ex. $|x|>5$



The solution set of $|x|>5$ contains all numbers whose distance from 0 is <u>more than</u> 5 units on the number line.

Absolute Value Inequalities of the Form $|X| < a$

If a is a positive number, then $|X| < a$ is equivalent to $-a < X < a$, where X is a variable or an expression.

This property also holds true for the inequality symbol $\leq$.

Ex. Solve. Write the solution in set-builder notation.

(a) $|y-2| \leq 5$

(b) $|x+1|+11 < 9$

(c) $-17+|3+5x| < -4$

(d) $|3x+6| \leq 0$

Absolute Value Inequalities of the Form $|X| > a$

If a is a positive number, then $|X| > a$ is equivalent to $X < -a$ or $X > a$, where X is a variable or an expression.

This property also holds true for the inequality symbol $\geq$.

Ex. Solve. Write the solution in set-builder notation.

(a) $|y-2| \geq 5$

(b) $|x+1| + 11 > 9$

(c) $-17 + |3+5x| > -4$

(d) $|3x+6| \geq 0$