

19.1 Radical Expressions and Functions

❖ Square Roots

Square Root

The number c is a square root of a if $c^2 = a$.

Ex. Find the square roots of 49.

Principal Square Root

The *principal square root* of a **nonnegative** number is its **nonnegative square root**.

Ex. The principal square root of 16 is 4.

Radical sign: $\sqrt{\quad}$

Radicand: the expression under the radical sign (ex. $\sqrt{5}$, \sqrt{a} , $-\sqrt{3x}$, $\sqrt{\frac{y^2+7}{y}}$)

Index (plural, **indices**): $\sqrt[n]{a}$ for n th root \rightarrow the number n is called the index.

Ex. Identify the radicand and the index for each expression.

(a) $-7\sqrt{y^2-8}$

(b) $a^2b^5\sqrt[3]{\frac{a}{a^2-b}}$

Ex. Simplify

(a) $\sqrt{196}$

(b) $-\sqrt{\frac{81}{144}}$

(c) $\sqrt{0.36}$

Square roots of perfect square radicands simplify to rational numbers. What happens when we try to simplify a root such as $\sqrt{11}$?

Use a calculator to approximate $\sqrt{11}$ to 3 decimal places. Check to see that your approximation is reasonable.

❖ Expressions of the Form $\sqrt{a^2}$

Simplifying $\sqrt{a^2}$

For any real number a ,

$$\sqrt{a^2} = |a|.$$

(The principal square root of a^2 is the absolute value of a .)

Ex. Simplify each expression. Assume that the variable can represent any real numbers.

(a) $\sqrt{25t^2}$

(b) $\sqrt{x^2 - 8x + 16}$

(c) $\sqrt{x^{16}}$

(d) $\sqrt{(r-1)^4}$

❖ Cube Roots

Cube Root

The number c is the cube root of a if $c^3 = a$. In symbols, we write $\sqrt[3]{a}$ to denote the cube root of a .

Ex. Simplify (a) $\sqrt[3]{64}$

(b) $-\sqrt[3]{-125y^6}$

❖ Odd and Even n th Roots $\sqrt[n]{a}$

Odd Roots

Odd roots of positive numbers are positive.

Odd roots of negative numbers are negative.

Absolute-value signs are not used when finding odd roots.

Even Roots

Even roots of positive numbers are positive.

Negative numbers **do not** have real n th roots when n is even.

Absolute-value signs are often necessary when finding even n th roots.

Ex. Simplify. Assume that the variables represent any real numbers. Remember to use absolute-value notation when necessary.

(a) $\sqrt[5]{-\frac{32}{243}}$

(b) $\sqrt[6]{(x-5)^6}$

(c) $\sqrt[4]{(-2)^4}$

Ex. Simplify. Assume that all variables represent nonnegative numbers.

(a) $\sqrt{\frac{y^{10}}{9x^6}}$

(b) $\sqrt[5]{-243a^{15}}$

(c) $\sqrt[4]{\frac{16m^4}{n^{20}}}$

❖ Radical Functions and Models

Radical Function: a function that can be describe by a radical expression.

Ex. For $g(x) = \sqrt{x^2 - 25}$, find the specified function value, if it exists.

(a) $g(-6)$

(b) $g(3)$

(c) $g(13)$

Ex. For $g(x) = -\sqrt[3]{2x-1}$, find the specified function value, if it exists.

(a) $g(-13)$

(b) $g(63)$

The square-root function: $f(x) = \sqrt{x}$ **Domain:** $[0, \infty)$ or $\{x \mid x \geq 0\}$

Graph $f(x) = \sqrt{x}$.

Ex. Determine the domain of each function described.

(a) $f(x) = \sqrt[4]{x-7}$

(b) $g(t) = \sqrt[3]{2t-5}$

(c) $H(x) = 5 - \sqrt[8]{1-4x}$

(d) $P(x) = 2 + \sqrt{3x-5}$