

19.2 Rational Numbers as Exponents

❖ RATIONAL EXPONENTS

Rational Exponents: An exponent that is a fraction.

Rational Exponents with a Numerator of 1

$$a^{1/n} = \sqrt[n]{a}$$

Ex. Write an equivalent expression using radical notation and, if possible, simplify.

(a) $(-8)^{1/3}$ (b) $-64^{1/2}$ (c) $\left(\frac{n^8}{36}\right)^{1/2}$

Ex. Write an equivalent expression using exponential notation.

(a) $\sqrt[3]{19}$ (b) $\sqrt[5]{3xy^2z}$

General Rule for Rational Exponents

For any natural numbers m and n ($n \neq 1$) and any real number a for which $\sqrt[n]{a}$ exists,

$$a^{m/n} \text{ means } \left(\sqrt[n]{a}\right)^m, \text{ or } \sqrt[n]{a^m}.$$

Ex. Write an equivalent expression using radical notation and, if possible, simplify.

(a) $-16^{5/4}$ (b) $(9y^6)^{3/2}$

(c) $\left(\frac{1}{32}\right)^{3/5}$ (d) $(5r-2)^{5/7}$ (e) $6a^{5/6}$

Ex. Write an equivalent expression using exponential notation.

(a) $(\sqrt[5]{8})^3$

(b) $\sqrt[9]{(2ab)^7}$

Negative Rational Exponents

For any rational number m/n and any nonzero real number a for which $a^{m/n}$ exists,

$$a^{-m/n} \text{ means } \frac{1}{a^{m/n}}.$$

Ex. Rewrite with positive exponents and, if possible, simplify.

(a) $-27^{-2/3}$

(b) $(-8)^{-4/3}$

(c) $\frac{1}{x^{-7/8}}$

(d) $\left(\frac{16x^4}{81}\right)^{-3/4}$

(e) $\frac{2a}{5b^{-1/3}}$

(f) $7p^{-6/5}q^{3/4}$

❖ USING RULES FOR EXPONENTS TO SIMPLIFY EXPRESSIONS

[Summary of Exponent Rules \(pg. 1251\)](#)

Ex. Use properties (laws) of exponents to simplify. Write results with only positive exponents.

(a) $5^{1/2} \cdot 5^{1/6}$

(b) $\frac{x^{3/4}}{x^{1/8}}$

(c) $(3^{2/9})^{-3/5}$

(d) $\left(\frac{ab^{-3/2}}{c^{-5/3}}\right)^{1/3}$

❖ USING RATIONAL EXPONENTS TO SIMPLIFY RADICAL EXPRESSIONS

Ex. Use rational exponents to simplify. Assume that variables represent positive numbers.

(a) $\sqrt[4]{36}$

(b) $\sqrt[8]{4y^2}$

(c) $\sqrt[9]{a^6b^3}$

(d) $(\sqrt[10]{x+1})^5$