19.3 Simplifying Radical Expressions

❖ Multiplying Radical Expressions

The Product Rule for Radicals

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$,

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}.$$

(To multiply, when the indices match, multiply the radicands.)

Ex. Multiply and simplify, if possible. Assume that all expressions under radicals represent nonnegative numbers.

(a) $\sqrt[3]{10} \cdot \sqrt[3]{4}$

(b) $\sqrt{50a^3b^2} \cdot \sqrt{2ab^4}$

(c) $\sqrt[4]{21r^3} \cdot \sqrt[4]{2r}$

(d) $3c\sqrt{15c^2} \cdot 2\sqrt{10c^5}$

❖ Simplifying by Factoring

Using the Product Rule to Simplify

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}.$$

Ex. Simplify. Assume that all expressions under radicals represent nonnegative numbers.

(a) $\sqrt{45}$

(b) $\sqrt[3]{-32a^6b}$

(c) $4\sqrt{50x^5}$

(d) $a^2b^3\sqrt[5]{a^{13}b^6c^{17}}$

❖ Dividing and Simplifying

The Quotient Rule for Radicals

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $b \neq 0$,

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}.$$

Ex. Simplify by taking roots of the numerator and the denominator. Assume that all expressions under radicals represent nonnegative numbers.

(a) $\sqrt{\frac{100}{81}}$

(b) $\sqrt{\frac{14}{x^2}}$

(c) $\sqrt[3]{\frac{64x^7}{216y^6}}$

(d) $\sqrt[4]{\frac{81x^4}{y^8z^4}}$

Ex. Divide and, if possible, simplify. Assume that all expressions under radicals represent nonnegative numbers.

(a) $\frac{\sqrt[3]{270}}{\sqrt[3]{10}}$

(b) $\frac{\sqrt{56ab^3}}{\sqrt{7a}}$

(c) $\frac{\sqrt[5]{64a^{11}b^{28}}}{\sqrt[5]{2ab^{-2}}}$

(d) $\frac{15\sqrt{24a^7b^3}}{5\sqrt{2a^2}}$

Ex. Simplify. Assume that all expressions under radicals represent nonnegative numbers.

(a) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^2}$

(b) $\dfrac{\sqrt[3]{x^2}}{\sqrt[5]{x}}$