19.8 The Complex Numbers

❖ Imaginary and Complex Numbers

Negative numbers **do not** have square roots in the **real-number system**.
Negative numbers **do** have square roots in the **complex-number system**.

The Number *i* (Imaginary Unit)
We define the number *i* such that $i = \sqrt{-1}$ and $i^2 = -1$.

Imaginary Numbers
An *imaginary number* is a number that can be written in the form bi, where b is a real number and $b \neq 0$.

Ex. Express in terms of *i*.

(a) $\sqrt{-36}$

(b) $-\sqrt{-75}$

(c) $\sqrt{-84}$

(d) $\sqrt{-18} - \sqrt{-100}$

Complex Numbers
A *complex number* is any number that can be written in the form $a + bi$, where a and b are real numbers. (Note that a and b both can be 0.)

❖ Addition and Subtraction

Ex. Add or subtract and simplify. Write each answer in the form $a + bi$.

(a) $(4 - 5i) + (3 + 9i)$

(b) $(-5 - i) - (7 + 4i)$

(c) $(14 - 7i) + (-8 - 9i)$

(d) $(5 - 2i) - (9 - 14i) + (16i)$

❖ Multiplication

Ex. Multiply and simplify. Write each answer in the form $a+bi$.

(a) $7i\cdot(-6i)$

(b) $\sqrt{-36}\cdot\sqrt{-9}$

(c) $5i(2+6i)$

(d) $(6-5i)(3+4i)$

❖ Dividing Complex Numbers

Ex. Divide: (a) $\dfrac{6-\sqrt{-27}}{3}$

(b) $\dfrac{24+\sqrt{-40}}{-8}$

Conjugate of a Complex Number

The *conjugate* of a complex number $a+bi$ is $a-bi$, and the conjugate of $a-bi$ is $a+bi$.

Ex. Multiply: $(3+i)(3-i)$

Ex. Divide and simplify to the form $a+bi$.

(a) $\dfrac{4}{3+i}$

(b) $\dfrac{2+7i}{5i}$