MATH 0310

Section 16.1 Functions and Graphs Supplement

Objective: Identify a relation as a function given (i) set of points, (ii) a table of values or (iii) an equation.

A **relation** is a set of ordered pairs. The set of all first elements of the ordered pair is called the **domain** of a relation, and the set of all second elements of the ordered pair is called the **range** of a relation.

For example: Find the domain and range of the relation {(0,2), (3,3), (-1,0), (3,-2)}.

> **Answer:** The domain is the set of all first elements of the ordered pair or {-1,0, 3} (note repeated values are only written once), and the range is the set of all second elements of the ordered pair, or {-2, 0, 2, 3}.

If we assume the domain consists of x-values and the range consists of y-values, then a function is a set of ordered pairs that assigns to each x-value **exactly one** y-value. This means, for example, we cannot have the ordered pairs (2, 3) and (2, 4) in the relation since the x-value does not have exactly one y-value.

Note it is okay for two different first elements to have the same second element. For example {(-2,4), (2,4), (-3,9), (3,9), (0,0)} is a function since each first element has exactly one second element. The domain is {-3, -2, 0, 2, 3} and the range is {0, 4, 9}.

You Try:

State the domain and range of each relation.

1. {(-1,0), (-1,6), (-1, 8)}
2. {(11,6), (-1,-2), (0,0), (3,-2)}

Answers:

1. Domain {-1} and Range {0, 6, 8}
2. Domain{-1, 0,3, 11} and Range {-2, 0, 6}

Determine whether each relation is also a function

1. {(-1,0), (-1,6), (-1, 8)}
2. {(11,6), (-1,-2), (0,0), (3,-2)}

Answers:

1. No
2. Yes

Determine whether a table of values represents a function.

Note a table of values is just an organized way of presenting the domain values and the range values of a relation. Thus the following table of values is the same as the relation {(2,4), (0,0), (-7,10), (10,-7)}. Does this table of values represent a function?

x	y
2	4
0	0
-7	10
10	-7

You Try:

Identify the domain and range of each relation and determine whether it is a function.

Answers:

1.

x	y
5	3
-3	3
7	7
-7	7

2.

Olympic Site	Year
Lake Placid	1980 1932
Calgary	1988
Squaw Valley	1960
Salt Lake City	2002

1. Domain {-7, -3, 3, 7}

   Range {3,7}

   Yes the relation is a function.

2. Domain {Lake Placid, Calgary, Squaw Valley, Salt Lake City}

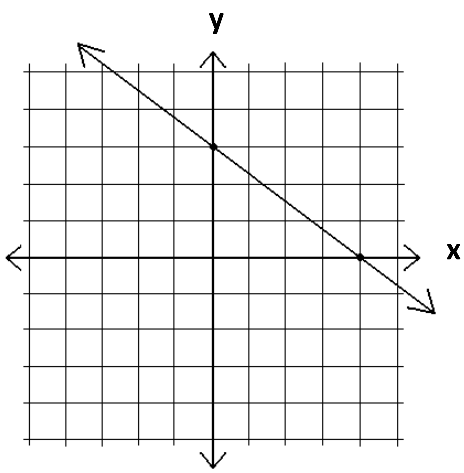
   Range {1932, 1960, 1980, 1988, 2002}

   No this relation is not a function.

Determine whether an equation represents a function.

Recall that the graph of a linear equation is a line. This means that equations of the form $y=mx+b$ and $y=b$ (where m and b are any real numbers) are functions since each x-coordinate will have exactly one y-coordinate.

For example: Does $y=-\frac{3}{4}x+3$ represent a function? We see from the graph below that each x-coordinate corresponds to exactly one y-coordinate. Thus this equation is a function.



The Vertical-Line Test
If it is possible for a vertical line to cross a graph more than once, then the graph is not the graph of a function.

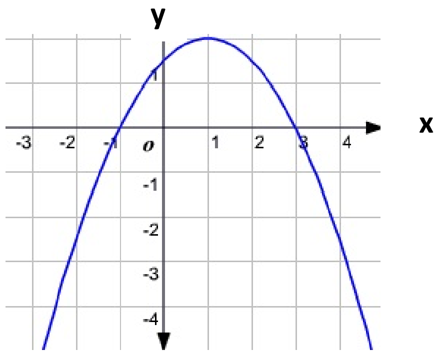
Note the graph above passes the Vertical-Line Test.

The domain of the graph is $(-\infty,\infty)$ and the range is also $(-\infty,\infty)$.

What about equations of the form $x=a$ (where a is any real number)? This graph is a vertical line, thus each x-coordinate does not have exactly one y-coordinate. In fact all of the x-coordinates are the same in this case and therefore there cannot be exactly one y-coordinate.

Note: Nonlinear equations can also be functions.

For instance if we look at the graph of y = $y = -\frac{1}{2}(x-1)^2 + 2$ it passes the Vertical-Line Test.



What about the equation $x = y^2$? Does this equation represent a function?

Let's suppose $x = 4$. There are two values of y that makes this true.

x	y
4	-2
4	2

What is the conclusion?

Is it safe to say that any equation containing y to a power is not a function?

Investigate with $x = y^3$.

What is your conclusion?

You Try:

Which of the following equations are functions?

1. $2x + 6y = 4$
2. $x = 3$
3. $y = x^2 - 4$
4. $2y - 3 = 6$
5. $y^2 = 2x + 5$

Answers:

1. Yes
2. No
3. Yes
4. Yes
5. No