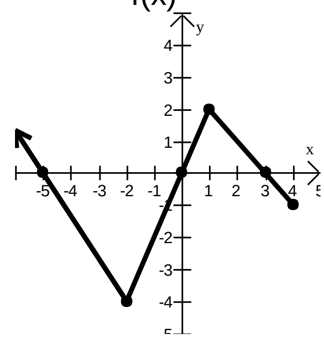
College Algebra Final Exam Review

For #1–4 use the given graph f(x):

f(x)



1.) Find f(–2).

2.) State the zeros, the domain, and the range.

3.) State the local maximum and/or minimum.

4.) State the intervals decreasing and increasing.

5.) State the domain of the following functions. Then determine whether each function is a one-to-one function.

a) f(x) = $\sqrt{3x-5}$ Is this function one-to-one?
b) f(x) = $2x^2+x+10$ Is this function one-to-one?
c) f(x) = $\dfrac{2}{3x+2}$ Is this function one-to-one?
d) f(x) = $e^{x+2}-3$ Is this function one-to-one?
e) f(x) = $\ln(x+4)$ Is this function one-to-one?

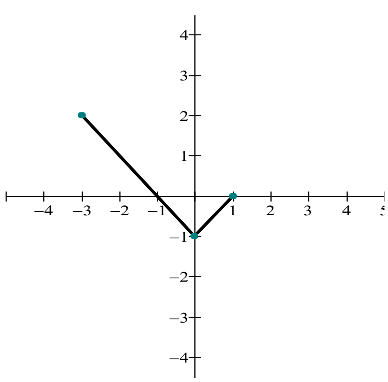
6.) Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for f(x) = 3x – 4.

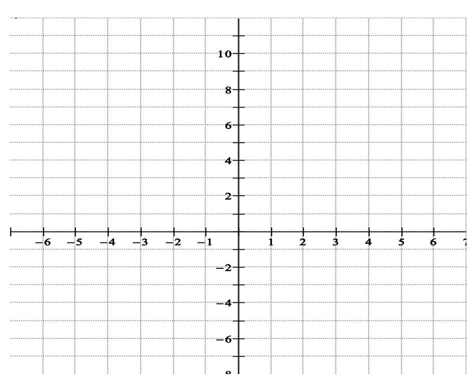
7.) Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for f(x) = $3x^2$ + 2x + 6.

8.) If the square root function is shifted 6 units to the right and 2 units down, what is the resulting function? State the domain.

9.) If the function f(x) = x^2 is shifted 7 units up and 3 units to the left, what is the resulting function? State the domain.

10.) For the function of f(x) shown below, use transformations to sketch the graph of f(x – 3) – 2.





11.) Graph $f(x)=\dfrac{1}{x-2}+3$ by applying transformations to $f(x)=\dfrac{1}{x}$. Give the equations of the asymptotes.

12.) Determine <u>algebraically</u> if the following functions are even, odd, or neither.
a) $f(x) = 3x^4 - 2x^2$ b) $g(x) = 2x^5 + x$ c) $h(x) = 7x^2 + 5x - 1$

13.) Let $f(x) = x^2 - 9x$ and $g(x) = 2x + 3$. Simplify completely.
a) Find $(f + g)(x)$ b) Find $(f - g)(x)$ c) Find $(fg)(x)$
d) Find $(f/g)(x)$ e) Find $(f \circ g)(x)$

14.) Find the vertex, y-intercept, and x-intercepts, if they exist, for $f(x) = x^2 - 2x - 35$.

15.) Solve $2x^2 + 16x + 26 = 0$.

16.) $P(x) = -x^2 + 90x - 300$ is a profit function where x is the number of items sold, and P(x) is the profit from that sale. Find the maximum profit and the number of items that must be sold to reach that profit.

17.) Analyze the polynomial $f(x) = 7(x - 4)^3(x + 1)^2(x + 3)$ and complete the chart below.

zeros	multiplicity (how many times it occurs)	Does the graph touch or cross at this intercept?

State the degree of the polynomial function. Determine the end behavior of the graph of the function.

18.) When a person gets a single flu shot, the concentration of the drug in milligrams per liter after t hours in the bloodstream is modeled by the following equation. Find the horizontal asymptote of the function, F(t), and interpret what the horizontal asymptote represents with respect to the concentration of flu medication in the bloodstream as time passes.

$$F(t) = \frac{7.6t}{t^2 + 0.2}$$

19.) For the rational function $R(x) = \frac{5x}{2x - 1}$ find the domain (D), equation of the vertical asymptote (VA), and equation of the horizontal asymptote (HA), if it exists. If non-existent, write NONE.

20.) Find all of the complex zeros for $f(x) = 2x^3 - 5x^2 - 23x - 10$. Also use the Rational Zeros Theorem to list the possible rational zeros.

21.) Find all of the complex zeros for $f(x) = x^4 - 7x^3 + 14x^2 + 2x - 20$. Also use the Rational Zeros Theorem to list the possible rational zeros.

22.) Find $f(-3)$ using Synthetic Division and the Remainder Theorem if $f(x) = 2x^3 - 5x^2 - 23x - 10$.

23.) Find a polynomial function that has as it zeros $x = -4$, $x = 1$, $x = 2i$, $x = -2i$. Write the polynomial function as a product of linear factors.

24.) The formula $S = \frac{7}{2}\sqrt{2D}$ can be used to approximate the speed S, in miles per hour, of a car that has left skid marks of length D, in feet. How far will a car skid at 70 mph? Round to one decimal place.

25.) The population P after t years of a newly introduced species of wildcat can be modeled by the equation $P(t) = \frac{900t + 8000}{3t + 4000}$.

a) Find the horizontal asymptote (HA).

b) Interpret this asymptote: As time, t, increases, the number of wildcats will approach and level off at __________.

26.) Find $f^{-1}(x)$ for $f(x) = 3x^3 + 2$ and $g(x) = \frac{x-8}{7}$.

27.) Write the following log expression as the sum and/or difference of logs with <u>no exponents or radicals remaining</u>: $\log_4\left(\frac{3\sqrt{x+2}}{4yz^3}\right)$.

28.) Solve $6 + e^{-4x} = 9$.

29.) Solve $3\log_2(8x) = 30$.

30.) Solve $\log_3(x + 1) - \log_3(x - 3) = \log_3(2)$.

31.) Solve: $64^x = 4^{2x+1}$.

32.) Simplify: $\log_{25}(5)$.

33.) Convert to a logarithmic equation: $4^x = 2.8$.

34.) Solve: $\log_2(x) + \log_2(x + 7) = 3$.

35.) Solve using Gauss-Jordan Elimination: $\begin{cases} x - 3y + 3z = 22 \\ 2x + 4y - z = -14 \\ 3x - y + 2z = 14 \end{cases}$

36.) Ron attends a cocktail party. He wants to limit his food intake to 133 g protein, 120 g fat, and 165 g carbohydrate. According to the health conscious hostess, the marinated mushroom caps have 3 g protein, 5 g fat, and 9 g carbohydrate; the spicy meatballs have 14 g protein, 7 g fat, and 15 g carbohydrate; and the deviled eggs have 13 g protein, 15 g fat, and 6 g carbohydrate. How many of each snack can he eat to obtain his goal? Solve using the Gauss-Jordan Elimination Method.

37.) If a couple needs \$14,500 for a down payment on a house and they invest the \$10,800 they have at 5.8%, compounded continuously, how long will it take for their money to grow to the \$14,500 needed?

38.) A radioactive substance decays according to the model $A(t) = A_0e^{-0.00472t}$, where A_0 is the initial amount present and t is the time in years.
a) If there are 20 grams present initially, when will there be 12 grams remaining?
b) In how many years will 80% of the original amount remain?

39.) For an arithmetic sequence with $a_1 = -22$ and $d = 3$,
a) Find a_{34} b) Find a formula for the nth term.

40.) For the arithmetic sequence 3, –1, –5, –9,....,
a) Find the common difference d b) Find a formula for the nth term d) Find a_{35}.

41.) For the geometric sequence with $a_1 = 3$ and $r = 2$,
a) Find a_{31} b) Find a formula for the nth term.

42.) For the geometric sequence 10, 2, $\frac{2}{5}, \frac{2}{25}, \ldots,$
a) Find a_{11} b) Find a formula for the nth term.

43.) Find the indicated sum of the arithmetic series: $\sum_{n=1}^{75}(25-3n)$.

44.) Find the indicated sum of the geometric series: $\sum_{n=1}^{12}(-2)^{n-1}$.

45.) If possible, find the sum of each geometric series: a) $\sum_{n=1}^{\infty}2\left(\frac{1}{2}\right)^{n-1}$ b) $\sum_{n=1}^{\infty}5\left(\frac{4}{3}\right)^{n-1}$.

46.) Consider the piecewise defined function $f(x) = \begin{cases} x^2+1 \text{ for } x<0 \\ 3x+2 \text{ for } x\geq 2 \end{cases}$.

a) Find f(–2), f(1), and f(5)

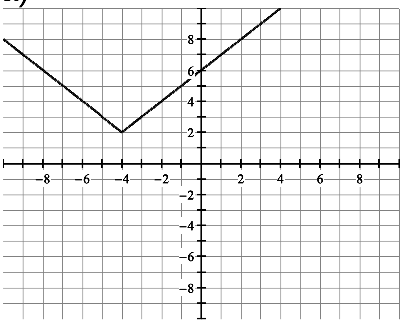
b) Graph this function.

47.) State the domain (D), range (R), & equation of the horizontal asymptote (HA) for each function.
a) $f(x) = e^{x+1}+5$ b) $g(x) = e^{-x} - 5$ c) $h(x) = -e^{x} - 5$.

48.) State the domain (D), range (R), & equation of the vertical asymptote (HA) for each function.
a) $f(x) = \log(x - 4)$ b) $g(x) = \ln(x + 4)$ c) $h(x) = -\log(x)$

49.) Which of the following graphs represents the graph of f(x) = |x + 4| + 2?

Which of the following graphs represents the graph of g(x) = –|x – 4| – 2?

a)


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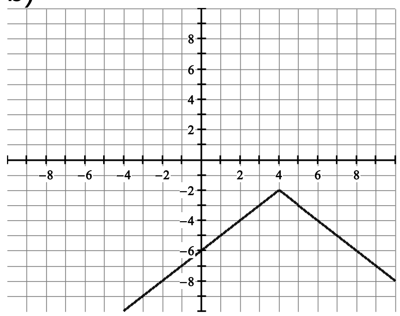

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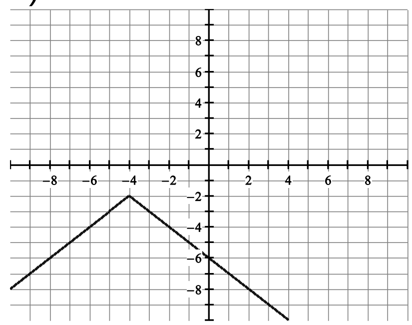

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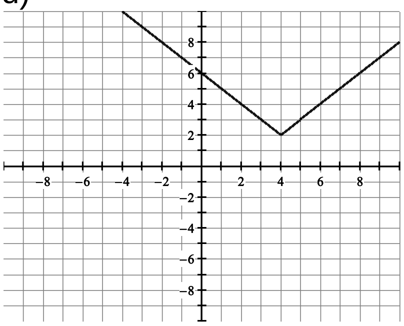

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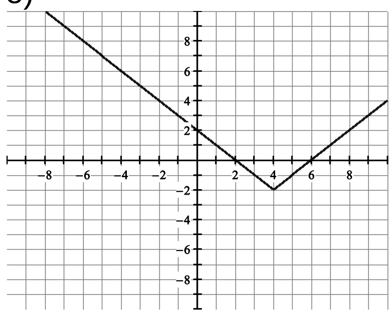
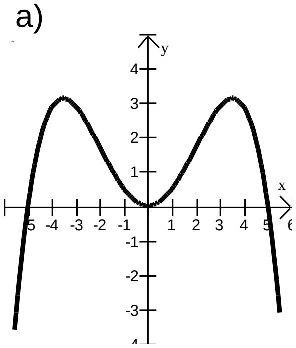


50.) Write the resulting matrix after the row operations have been applied:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & -5 & 1 \\ -3 & -2 & 1 & 7 \end{array}\right] \qquad \begin{array}{l} 3R_1 + R_3 \rightarrow R_3 \\ \frac{1}{5}R_2 \rightarrow R_2 \end{array}$$

51.) State the domain and range of each graph. Determine whether each function is a one-to-one function.

a)


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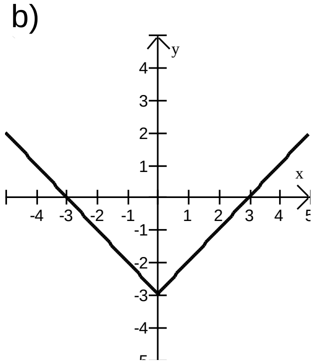
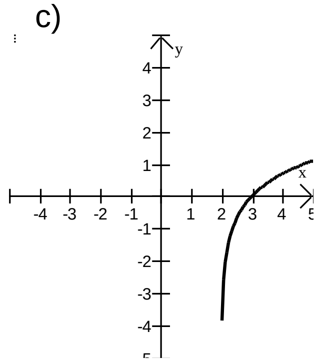


c)


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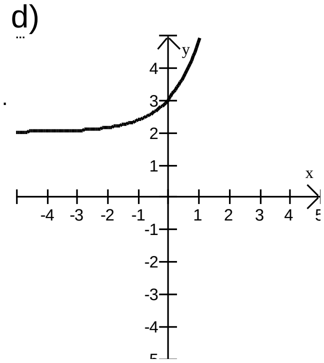

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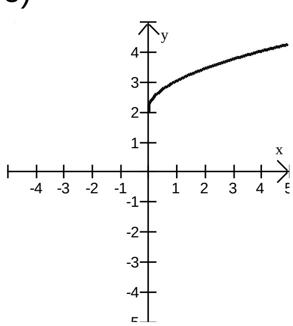

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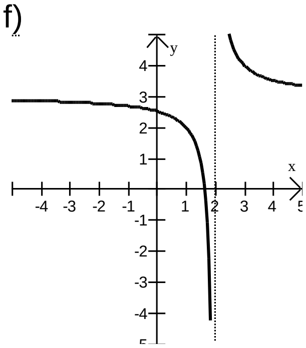


52.) Find the 1st 4 terms of the sequence whose n^{th} term is $a_n = n^2 + 1$.

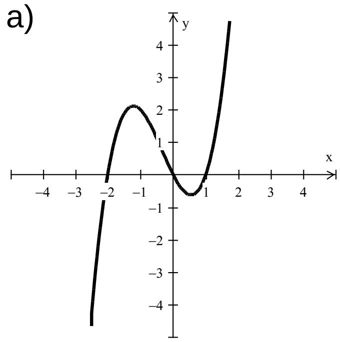
53.) Find and evaluate: $\sum_{k=1}^{3}(k^2)$.

54.) Suppose that a polynomial function of degree 5 with rational coefficients has 4, –2i, and 4 + i as zeros. Find the remaining zeros.

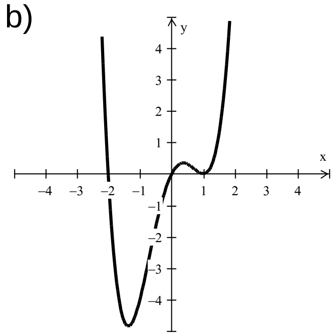
55.) Write as a single logarithm: log(3) + 2log(x) – log(5)

56.) For each polynomial function graphed, determine the minimum possible degree, the zeros and if the multiplicity of the zeros is even or odd.

a)



b)



Answers

1.) –4

2.) –5, 0, 3; Domain: (–∞, 4]; Range: [–4,∞)

3.) Local max: 2 when x = 1
Local min: –4 when x = –2

4.) Intervals decreasing: (–∞, –2) U (1, 4)
Interval increasing: (–2, 1)

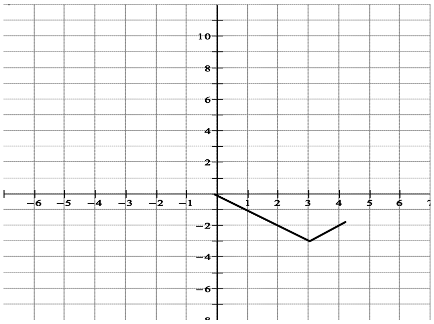
5.) a) $\left[\frac{5}{3},\infty\right)$; yes b) (–∞, ∞); no c) $\left(-\infty,-\frac{2}{3}\right)\cup\left(-\frac{2}{3},\infty\right)$; yes
d) (–∞, ∞); yes e) (–4, ∞); yes

6.) 3

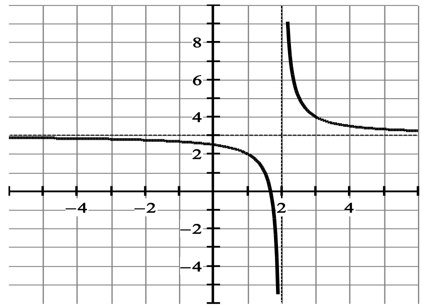
7.) 6x + 3h + 2

8.) $f(x)=\sqrt{x-6}-2$; Domain: [6,∞)

9.) f(x) = $(x + 3)^2$ + 7; Domain: (–∞, ∞)

10.) 

11.) VA: x = 2 HA: y = 3



12.) Recall the algebraic definitions of even & odd:
g(–x) = g(x) => g is even => symmetry about the y-axis
g(–x) = –g(x) => g is odd => symmetry about the origin

a) Even: Show that f(–x) = f(x)
$f(x) = 3x^4 - 2x^2$
$f(-x) = 3(-x)^4 - 2(-x)^2 = 3x^4 - 2x^2$

b) Odd: Show that –g(x) = g(–x)
$g(x) = 2x^5 + x$
$g(-x) = 2(-x)^5 + (-x) = -2x^5 - x$
$-g(x) = -(2x^5 + x) = -2x^5 - x$

c) Neither: Show that h(–x) does not equal h(x) and h(–x) does not equal –h(x)
$h(x) = 7x^2 + 5x - 1$
$h(x) = 7(-x)^2 + 5(-x) - 1 = 7x^2 - 5x - 1$
$h(x) = -(7x^2 + 5x - 1) = -7x^2 - 5x + 1$

13.) a) $(f + g)(x) = x^2 - 7x + 3$ b) $(f - g)(x) = x^2 - 11x - 3$ c) $(fg)(x) = 2x^3 - 15x^2 - 27x$
d) $(f/g)(x) = \dfrac{x^2 - 9x}{2x + 3}$ e) $(f \circ g)(x) = 4x^2 - 6x - 18$

14.) Vertex: (1, –36); y-intercept: (0, –35); x-intercepts: (–5,0) and (7, 0)

15.) $x = -4 \pm \sqrt{3}$

16.) maximum profit: $1725; number of items: 45

17.) zero: 4 has a multiplicity of 3, crosses
zero: –1 has a multiplicity of 2, touches
zero: –3 has a multiplicity of 1 crosses
degree: 6 end behavior: up left and up right

18.) HA: y = 0; As time passes (t increases), the concentration of flu medication in the bloodstream approaches 0.

19.) D: $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$; VA: $x = \dfrac{1}{2}$; HA: $y = \dfrac{5}{2}$

20.) zeros: –2, $-\dfrac{1}{2}$, 5 possible rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10, \pm \dfrac{1}{2}, \pm \dfrac{5}{2}$

21.) zeros: –1, 2, 3 + i, 3 – i possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

22.) –40; synthetic division:

$$\begin{array}{r|rrrr} -3 & 2 & -5 & -23 & -10 \\ & & -6 & 33 & -30 \\ \hline & 2 & -11 & 10 & \underline{|-40} \end{array}$$

The remainder is –40, so f(–3) = –40.

23.) $f(x) = (x + 4)(x - 1)(x - 2i)(x + 2i)$

24.) 200 feet

25.) HA: $y = 300$; 300 wildcats

26.) $f^{-1}(x) = \sqrt[3]{\dfrac{x-2}{3}}$ $\quad g^{-1}(x) = 7x + 8$

27.) $\log_4(3) + \frac{1}{2}\log_4(x + 2) - 1 - \log_4(y) - 3\log_4(z)$

28.) $x = -\dfrac{\ln(3)}{4} \approx -0.2747$

29.) $x = 128$

30.) $x = 7$

31.) $x = 1$

32.) $x = \dfrac{1}{2}$

33.) $x = \log_4(2.8)$

34.) $x = 1$

35.) Solution: (1, –3, 4); Set of Row Operations:

$-2R_1 + R_2 \rightarrow R_2$
$-3R_1 + R_3 \rightarrow R_3$
$\frac{1}{10}R_2 \rightarrow R_2$
$3R_2 + R_1 \rightarrow R_1$
$-8R_2 + R_3 \rightarrow R_3$
$-\frac{5}{7}R_3 \rightarrow R_3$
$-\frac{9}{10}R_3 + R_1 \rightarrow R_1$
$\frac{7}{10}R_3 + R_2 \rightarrow R_2$

36.) # of mushrooms caps = 8 # of meatballs = 5 # of deviled eggs = 3

37) 5.08 years

38.) a) in 108 years b) in 47.3 years

39.) $a_{34} = 77$; $a_n = 3n - 25$

40.) $d = -4$; $a_n = -4n + 7$; $a_{35} = -133$

41.) $a_{31} = 3{,}221{,}225{,}472$; $a_n = 3(2)^{n-1}$

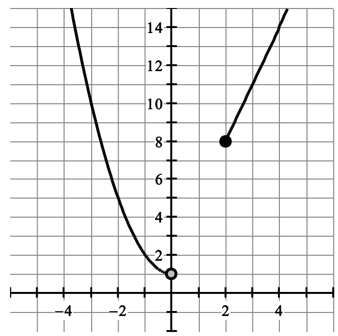
42.) $a_{11} = 1.024 \times 10^{-6}$; $a_n = 10\left(\frac{1}{5}\right)^{n-1}$

43.) –6675

44.) –1365

45.) a) 4 b) The sum does not exist.

46.) a) $f(-2) = 5$, $f(1)$ is undefined, $f(5) = 17$ b)



47.) a) D: $(-\infty, \infty)$; R: $(5, \infty)$; HA: $y = 5$
b) D: $(-\infty, \infty)$; R: $(-5, \infty)$; HA: $y = -5$
c) D: $(-\infty, \infty)$; R: $(-\infty, -5)$; HA: $y = -5$

48.) a) D: $(4, \infty)$; R: $(-\infty, \infty)$; VA: $x = 4$
b) D: $(-4, \infty)$; R: $(-\infty, \infty)$; VA: $x = -4$
c) D: $(0, \infty)$; R: $(-\infty, \infty)$; VA: $x = 0$

49.) a); b)

50.) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 1/5 \\ 0 & 4 & 10 & 19 \end{array}\right]$

51.)

a) $(-\infty, \infty)$	b) $(-\infty, \infty)$	c) $(2, \infty)$	d) $(-\infty, \infty)$	e) $[0, \infty)$	f) $(-\infty, 2) \cup (2, \infty)$
$(-\infty, 3]$	$[-3, \infty)$	$(-\infty, \infty)$	$(2, \infty)$	$[2, \infty)$	$(-\infty, 3) \cup (3, \infty)$
No	No	Yes	Yes	Yes	Yes

52.) 2, 5 10, 17

53.) 14

54.) 2i, 4 – i

55.) $\log\left(\frac{3x^2}{5}\right)$

56.) a) Degree 3; zeros: –2 (odd multiplicity), 0 (odd multiplicity), 1 (odd multiplicity)
b) Degree: 4; zeros: –2 (odd multiplicity), 0 (odd multiplicity), 1 (even multiplicity)