4.4 Properties of Logarithms

Properties of exponents correspond to properties of logarithms.
(logarithm = exponent)

Logarithmic Properties (p.445)

For $M > 0$ and $N > 0$:

The Product Rule	$\log_b(MN) = \log_b M + \log_b N$	(Sum of Logs)
The Quotient Rule	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$	(Diff. of Logs)
The Power Rule	$\log_b M^p = p\log_b M$	(Prod. of Expo. & Log)

Ex. Use properties of logarithms to **expand** each logarithmic expression as much as possible. Write the expression as the sum or difference of logarithms. Where possible, evaluate logarithmic expressions without using a calculator.

a.) $\log_9(9x) =$

b.) $\log\left(\frac{x}{1000}\right) =$

c.) $\log_b(xy^3) =$

d.) $\log_8\left(\frac{64}{\sqrt{x+1}}\right) =$

e.) $\ln \sqrt[5]{\frac{x}{y}} =$

f.) $\log\left(\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right) =$

Ex. Use properties of logarithms to **condense** each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions without using a calculator.
[Note: Coefficients of logarithms must be 1 before you can condense them using the product and quotient rules.]

a.) $\log x + 7\log y$

b.) $4\ln x + 7\ln y - 3\ln z$

c.) $\frac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$

d.) $\frac{1}{3}\left[5\ln(x+5) - \ln x - \ln\left(x^2-25\right)\right]$

e.) $\log x + \log\left(x^2 - 4\right) - \log 15 - \log(x+2)$

❖ **The Change-of-Base Property**

$$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

(evaluate logs that have a base other than 10 or *e*)

Ex. Use common logarithms or natural logarithms and a calculator to evaluate to four decimal places. Check the result by using the related exponential form.

a.) $\log_6 17$

b.) $\log_{0.3} 19$

Ex. Determine whether each equation is true or false. Where possible, show work to support your conclusion. If the statement is false, make the necessary change(s) to produce a true statement.

a.) $\ln\left(8x^3\right) = 3\ln(2x)$

b.) $\ln x + \ln(2x) = \ln(3x)$

c.) $\dfrac{\log(x+2)}{\log(x-1)} = \log(x+2) - \log(x-1)$