2.7 Analyzing Graphs of Functions and Piecewise-Defined Functions

❖ **Even and Odd Functions**

Algebraic Test:

A function is an **even function** if $f(-x) = f(x)$ for all x in the domain of the function.

A function is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of the function.

Ex. Determine whether each of the following functions is even, odd, or neither algebraically:

  a) $f(x) = x^3 - x$

  b) $f(x) = x^2 - x$

  c) $f(x) = 2x^2 + x^4$

Graphical Test:

An **even function** is one that is symmetric with respect to the **y-axis**.
If (a, b) is on the graph, the point $(-a, b)$ is also on the graph.
An **odd function** is one that is symmetric with respect to the **origin**.
If (a, b) is on the graph, the point $(-a, -b)$ is also on the graph.

Ex. Use a graphing calculator to determine whether each of the following functions is even, odd, or neither:

  a) $f(x) = x^2 - 3$

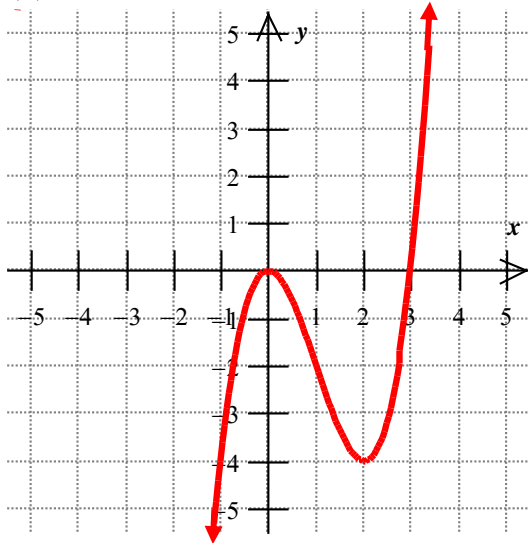 b) $f(x) = 7x(x^2 - 1)$

  c) $f(x) = -2(x+3)^2 + 5$

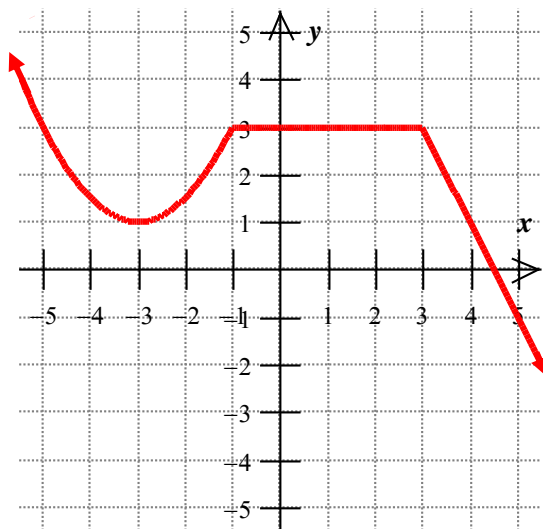## ❖ Increasing, Decreasing, and Constant Functions

This is an algebraic description for <u>the behavior of a function</u> with respect to the value along the *x*-axis.

Ex. State the intervals where the graph is increasing, decreasing, and constant.
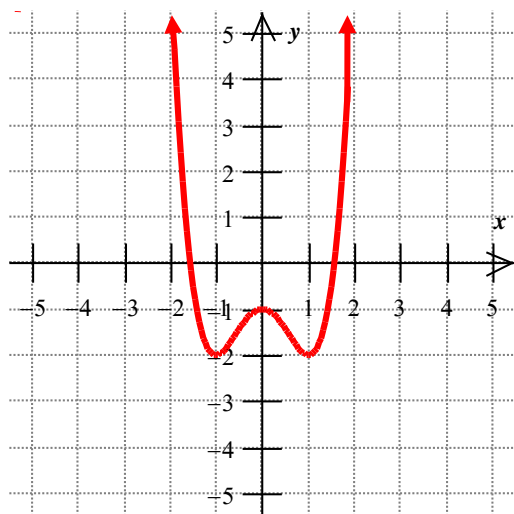
(a)

(b)

❖ Relative (Local) Maxima and Relative (Local) Minima

<u>**Relative Maxima:**</u> **high points** in relation to the rest of the graph
<u>**Relative Minima:**</u> **low points** in relation to the rest of the graph
These are where the graph changes direction from increasing to decreasing or vice versa.

Ex. The graph of a function f is given. Use the graph to find each of the following:

(a) The function f has a relative maximum at

$x =$ _____ .

The relative maximum is _____ .

(b) The function f has a relative minimum at

$x =$ _____ .

The relative minimum is _____ .

❖ Piecewise Functions
<u>**Piecewise Function:**</u> a function that is defined by two (or more) equations over a specified domain.

Ex. A cellular phone company offers the following plan:
- $40 per month buys 200 minutes.
- Additional time costs $0.30 per minute.

(a) Write the total monthly cost, C, as a function of the number of calling minutes, t.

(b) What is the total monthly cost when the customer used 120 minutes?

(c) What is the total monthly cost when the customer used 250 minutes?

Ex. Evaluate the function for the given values of x.

$$f(x) = \begin{cases} 1 & \text{if } x < -3 \\ -x & \text{if } -3 \le x < 0 \\ x^2 - 1 & \text{if } x \ge 0 \end{cases}$$

(a) $f(-5)$

(b) $f(-3)$

(c) $f(-1)$

(d) $f(0)$

(e) $f(2)$

(f) $f(3)$

Ex. Graph the following piecewise function:

$$f(x) = \begin{cases} 1 & \text{if } x < -3 \\ -x & \text{if } -3 \le x < 0 \\ x^2 - 1 & \text{if } x \ge 0 \end{cases}$$