2.8 Algebra of Functions and Function Composition

❖ **The Domain of a Function (REVIEW)**

Domain: The set of all input values (*x*-values) for which the value of $f(x)$ (*y*-values) is a real number.

Finding a Function's Domain:

   1.) The domain is the set of all real numbers, $(-\infty, \infty)$, unless x appears in a denominator or a square root.

   2.) "Fraction" – the denominator can NOT equal 0.

       ➔ Set the **expression** in the denominator **= 0** and solve.

   3.) Square Root – radicand must be greater than or equal to 0.

       ➔ Set the **expression** under the radical sign, $\sqrt{}$, ≥ 0 and solve.

Ex. Find the domain of each function in interval notation.

(a) $f(x) = x^2 + x - 12$

(b) $f(x) = \dfrac{x^3}{5}$

(c) $f(x) = \dfrac{5}{x^2 - 2x - 15}$

(d) $f(x) = \dfrac{1}{\dfrac{4}{x-2} - 3}$

(e) $f(x) = \sqrt{5x - 8}$

(f) $f(x) = \dfrac{3x}{\sqrt{x+8}}$

❖ The Algebra of Functions

If f and g are functions and x is in the domain of both functions, then:

1. **Sum:** $(f+g)(x) = f(x) + g(x) = f + g$;

2. **Difference:** $(f-g)(x) = f(x) - g(x) = f - g$;

3. **Product:** $(f \cdot g)(x) = f(x) \cdot g(x) = f \cdot g$;

4. **Quotient:** $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, $\quad g(x) \neq 0 = \dfrac{f}{g}$, $\quad g \neq 0$.

Ex. Let $f(x) = x^2 + 4x - 12$ and $g(x) = x - 2$. Find $f + g$, $f - g$, $f \cdot g$, and $\dfrac{f}{g}$.

Determine the domain for each function.

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $(f \cdot g)(x)$

(d) $\left(\dfrac{f}{g}\right)(x)$

Hint: If the function $\dfrac{f}{g}$ can be simplified, determine the domain **before** simplifying.

Ex. Let $f(x) = 6 - \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x}$. Find each of the following:

(a) $\left(\dfrac{f}{g}\right)(x)$

(b) The domain of $\left(\dfrac{f}{g}\right)(x)$

Ex. Let $f(x) = -2x + 3$ and $g(x) = |x + 4|$. Find

(a) $(f + g)(0)$

(b) $(f - g)(-3)$

(c) $(f \cdot g)(2)$

(d) $\left(\dfrac{f}{g}\right)(6)$

❖ **Difference Quotient of a Function**

$$\dfrac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

Steps of finding difference quotient of a function:

Step 1: Find $f(x+h)$.

Step 2: Subtract $f(x)$ from $f(x+h)$

Step 3: Divide by h

Ex. Find the difference quotient if $f(x) = 6x + 1$.

Ex. Find the difference quotient if $f(x) = x^2 - 5x + 8$.

❖ **Composite Functions**

The composite function $f \circ g$, the composition of f and g, is defined as
$$(f \circ g)(x) = f(g(x)).$$

$f \circ g$ read "the composition of f and g,"
  "f composed with g,"
  "f circle g"

"plug the g function (the second function) into the f function (the first function)"

Ex. Given $f(x) = 3x^2 - 4$; $g(x) = 2x + 5$

Find (a) $(f \circ g)(-2)$ (b) $(g \circ f)(-2)$

(c) $(f \circ g)(x)$ (d) $(g \circ f)(x)$